$$n_j^{FD} = \frac{g_j}{e^{\alpha+\varepsilon_j/kT}+1} \qquad n_j^{BE} = \frac{g_j}{e^{\alpha+\varepsilon_j/kT}-1} \qquad \text{(IIIB-14,16)}$$

We begin with applications to waves.

I. PROPERTIES OF WAVES

Before we can apply eqn. IIIB-16 to determine the properties of systems of massless Bosons (waves), we must evaluate the parameter α. Recall that two unknown multipliers, α and β, were introduced as a step in Lagrange's method of finding a maximum subject to imposed conditions. The problem was to maximize the entropy, or $\ln W_i$, for an isolated system.

$$d\left(\frac{S}{k} - \alpha N - \beta U\right) = d\left[\ln W(n_{j,i}) - \alpha \sum_j \delta n_{j,i} - \beta \sum \varepsilon_j \delta n_{j,i}\right] = 0$$

The parameter β prescribes conservation of energy. After the three equilibrium distribution functions were derived in terms of the unknown multipliers, α and β, the concept of thermal equilibration was employed to evaluate β in general, $\beta \equiv 1/kT$ (see Exmps. 2 and B4 and Exer. B4 of Unit IIIB). The parameter α, on the other hand, is intended to prescribe conservation of mass. In fact, however, it prescribes conservation of particles. In the case of particles of nonzero mass, the two conservation principles are the same. But for systems of massless particles, mass is conserved independent of the number of particles present. Indeed photons and phonons can be created or destroyed subject only to conservation of energy. Thus, the multiplier α is unnecessary for systems of massless particles. (That is, $\alpha = 0$.)[3] Thus, *the distribution function for massless particles becomes*

$$n_j^o = \frac{g_j}{e^{\varepsilon_j/kT}-1} \qquad \text{(IIID-1a)}$$

Note to the Student. The following exercise is not an objective of the unit. It provides the interested student an opportunity to verify eqn. IIID-1 through a more formal derivation.

Exercise 2. Modify the derivation of eqn. IIIB-16 as completed in Exer. B2, Appendix B of Unit IIIB to exclude the conservation of particles requirement. Verify that eqn. IIID-1 is the result. (No answer to this exercise is presented.)

In the following sections we employ eqn. IIID-1 to develop the properties of electromagnetic and acoustical radiation.

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[3]This verifies that massless particles do not satisfy the Boltzmann limit. As $n_j/g_j|_{max} << 1$ requires that $e^{\alpha} >> 1$, $\alpha = 0$ corresponds to a system which is always degenerate, viz., never satisfies the Boltzmann limit.