Thus,

$$\left(\frac{\partial S}{\partial V}\right)_U = \frac{\partial}{\partial V}\left(\frac{4}{3}\,VaT^3\right)_U = \frac{4}{3}\,aT^3 + \frac{4}{3}\,Va3T^2\left(\frac{\partial T}{\partial V}\right)_U$$

The last term can be evaluated using

$$U = VaT^4 \qquad V = \frac{U}{aT^4} \qquad \left(\frac{\partial V}{\partial T}\right)_U = -\frac{4U}{aT^5} \qquad \left(\frac{\partial T}{\partial V}\right)_U = -\frac{aT^4}{4U} = -\frac{T}{4V}$$

Then

$$\left(\frac{\partial S}{\partial V}\right)_U = \frac{4}{3}\,aT^3 + \frac{4}{3}\,Va3T^2\left(-\frac{T}{4V}\right) = \frac{aT^3}{3} = \frac{u_v}{3T}$$

Exmps. 3 and 4 derived expressions for the photon entropy density and radiation pressure, respectively. Both results can be written in terms of the photon energy density.

$$s_{v,pht} = \frac{4}{3}\,\frac{u_{v,pht}}{T} \qquad p_{pht} = \frac{u_{v,pht}}{3} \qquad \text{(IIID-9,10)}$$

Exercise 8. Determine the number of photons per cubic meter and the photon energy and entropy per cubic meter and the radiation pressure at 300, 3000, and 30,000 K.

The photon gas is always present. Thus, its properties should be added to those of the particles of matter to obtain total properties. In that respect our calculation of perfect gas properties, Unit IIIC, was incomplete. The following problem compares the photon and molecular contributions to perfect gas properties.

Exercise 9. Both photon density and photon energy density are functions of temperature only. Gaseous particle density and therefore gaseous energy density are dependent on both pressure and temperature. To compare the separate property contributions of photons and molecules, determine the pressure level at which molecular density would equal the photon density at 300, 3000, and 30,000 K. In addition, determine the pressure level at which the translational component of molecular energy would equal the photon energy per unit volume. (Use the results of Exer. 8.)

The results of Exer. 9 demonstrate that the property contributions of photons are an appreciable part of the thermostatic properties of a gas only at extremely high tempera-

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Ans. 6. Using $x = (hc/\lambda kT)$, $dx = -(hc/\lambda^2 kT)d\lambda$ and $\lambda = 0$, $x = \infty$; $\lambda = \infty$, $x = 0$, we have

$$n_{v,pht} = \frac{8\pi V}{V}\int_0^\infty \frac{d\lambda}{\lambda^4[e^{(hc/\lambda kT)}-1]} = \pi\left(\frac{kT}{hc}\right)^3\int_0^\infty\left(\frac{hc}{\lambda kT}\right)^2\left(\frac{hc}{\lambda^2 kT}d\lambda\right)\frac{1}{[e^{(hc/\lambda kT)}-1]} = 8\pi\left(\frac{kT}{hc}\right)^3\left[\int_0^\infty \frac{x^2dx}{(e^x-1)}\right]$$

This integral is evaluated in terms of the Gamma Function and the Riemann Zeta Function, see Ref. 5. The result is expressed as eqn. IIID-8.

Ans. 7. The internal energy per mole of photons is

$$\bar{u}_{pht}(T) = \frac{u_v(T)N_A}{n_v(T)} = \frac{\frac{4}{c}\left(\frac{2\pi^5k^4}{15\,h^3c^2}\right)T^4\,N_A}{8\pi\cdot(2.404)\cdot\left(\frac{k}{hc}\right)^3\,T^3} = \frac{\pi^4\bar{R}T}{36.06}. \qquad \text{Thus} \qquad \bar{c}_{v,pht} = \left(\frac{\partial\bar{u}_{pht}(T)}{\partial T}\right)_v = \frac{\pi^4}{36.06}\bar{R}$$