process does allow us to achieve considerable insight with respect to how K_{phn} varies with temperature, see Fig. IIIE-3.

The Thermal Conductivity of Insulating Solids. Heat transport within an opaque electrical nonconductor is dominated by phonon transport. The thermal conductivity in such crystals is predicted by eqn. IIIE-20a, where the effective mean free path is determined from

$$\frac{1}{\lambda_{c,phn}} = \frac{1}{\lambda_{c,phn-phn}} + \frac{1}{\lambda_{c,phn-imp}}$$

Thus, phonon heat transport is limited by the most efficient collison process, i.e., the shortest mean free path. This limiting process changes as we go from one region of temperature to another.

The *high temperature region* is defined by $T > \theta_D$. In this range, $c_{v,phn} = 3R$ in atomic lattices and 5R or 6R in molecular lattices. Also the phonon density is high enough that phonon-phonon collisions dominate; $n_{v,phn} \propto T$ (see Unit IIID) and $\lambda_{c,phn-phn} << \lambda_{c,phn-imp}$. Thus thermal conductivity is inversely proportional to temperature.

$$K_{ins}(T > \theta_D) \propto \frac{\rho c_s (3)R}{3n_{v,phn}} \propto \frac{1}{T}$$

The *low temperature region*, $T << \theta_D$. At low temperatures $c_{v,phn} \quad T^3$ and phonon-impurity collisions will dominate $\lambda_{c,phn-phn} > \lambda_{c,phn-imp} \sim \delta L$. Thus the conductivity will be proportional to

$$K_{ins}(T << \theta_D) \propto \frac{\rho c_s \delta L \; T^3}{3} \propto T^3$$

The intermediate temperature region, $T \sim \theta_D$. The temperature variation of K_{ins} in this region is strongly dependent on the purity and quality of the crystal. In very pure, carefully grown single crystals $\lambda_{c,phn-imp}$ approaches the size of the crystal and phonon-phonon collisions continue to dominate to quite low temperatures. This gives relatively large values of K_{ins} down to quite low temperatures. In ordinary solids with an appreciable impurity concentration, $\lambda_{c,phn-imp}$ is much shorter and these processes will become dominant at higher temperatures. Below this point, the conductivity follows the Debye specific heat curve.

$$K_{ins}(T \sim \theta_D) \propto \frac{\rho c_s \delta L \; c_{v,phn}(T)}{3} \propto c_{v,phn}(T)$$

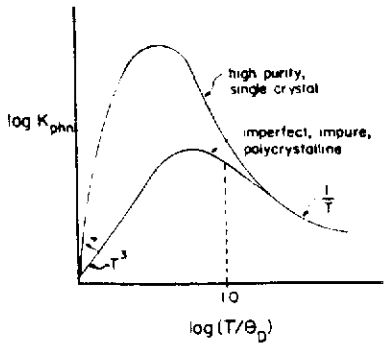

Fig. IIIE-3. Insulator Thermal Conductivities (Schematic)

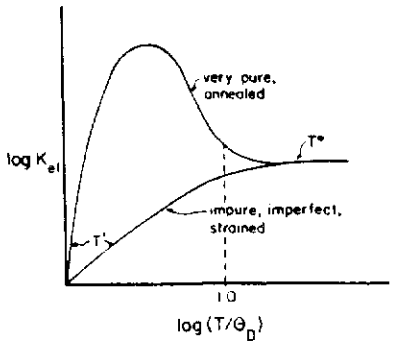

Fig. IIIE-4. Metallic Thermal Conductivities (Schematic)