$$S_{eq}^{FD} = k\beta U + Nk\alpha + k \sum_j g_j \ \ell n \left[1 + e^{-\alpha-\beta\varepsilon_j(V)}\right]$$

Example B4. Use the form of S_{eq}^{FD} developed in Exmp. B1 to show that $(\partial S_{eq}^{FD}/\partial U)_{N,V} = k\beta$. The terms in the partial derivative are[21]

$$\left(\frac{\partial S_{eq}^{FD}}{\partial U}\right)_{N,V} = k\beta + kU\left(\frac{\partial\beta}{\partial U}\right)_{N,V} + Nk\left(\frac{\partial\alpha}{\partial U}\right)_{N,V} + \sum_j \frac{-\varepsilon_j g_j e^{-\alpha-\beta\varepsilon_j(V)}}{1+e^{-\alpha-\beta\varepsilon_j(V)}} \cdot \left(\frac{\partial\beta}{\partial U}\right)_{N,V}$$

$$+ k \sum_j \frac{-g_j e^{-\alpha-\beta\varepsilon_j(V)}}{1+e^{-\alpha-\beta\varepsilon_j(V)}} \cdot \left(\frac{\partial\alpha}{\partial U}\right)_{N,V}$$

Then recognize that

$$n_{j,mp}^{FD} = \frac{g_j}{[e^{\alpha+\beta\varepsilon_j(V)}+1]} = \frac{g_j e^{-\alpha-\beta\varepsilon_j(V)}}{[1+e^{-\alpha-\beta\varepsilon_j(V)}]}$$

And substitute to obtain

$$\left(\frac{\partial S_{eq}^{FD}}{\partial U}\right)_{N,V} = k\beta + k\left(\frac{\partial\beta}{\partial U}\right)_{N,V}\left[U - \sum_j n_{j,mp}^{FD}\ \varepsilon_j(V)\right] + k\left(\frac{\partial\alpha}{\partial U}\right)_{N,V}\left(N - \sum_j n_{j,mp}^{FD}\right) = k\beta$$

A review of the identification of β in Section II will reveal that $(\partial S_{eq}/\partial U)_{N,V} = k\beta$ is sufficient to obtain the result. Thus $\beta = 1/kT$ is valid for Fermions. The following exercises obtain analogous results for Bosons. We thus obtain the equilibrium distribution functions and entropies of Fermions and Bosons as

$$n_{j,mp}^{FD} = \frac{g_j}{e^{\alpha+\varepsilon_j(V)/kT}+1} \qquad n_j^{BE} = \frac{g_j}{e^{\alpha+\varepsilon_j(V)kT}-1} \qquad \text{(IIIB-B2,-B4)}$$

$$S_{eq}^{FD} = \frac{U}{T} + Nk\alpha + k \sum_j g_j \ \ell n \left[1+e^{-\alpha-\varepsilon_j(V)/kT}\right] \qquad \text{(IIIB-5)}$$

$$S_{eq}^{BE} = \frac{U}{T} + Nk\alpha - k \sum_j g_j \ \ell n \left[1-e^{-\alpha-\varepsilon_j(V)/kT}\right] \qquad \text{(IIIB-6)}$$

Ans. B1. The logarithm of eqn. IIIA-7b by the product rule is

$$\ell n\ W_i^{BE} = \sum_j \{\ell n\ [(g_j+n_{j,i}-1)!] - \ell n\ [(g_j-1)!] - \ell n\ (n_{j,i}!)\}$$

Then, since $g_j >> 1$ and by $\ell n\ x! \simeq x\ \ell n\ x - x$, we have