Example 7. The following are examples of classification of indistinguishable particles as symmetric (even) or antisymmetric (odd) and the selection of an appropriate statistic.

(i) The common isotope of helium, He^4, has two electrons, two protons, and two neutrons. Its total number of subatomic components is 6—even, therefore He^4 particles are symmetric particles, Bosons. The statistics applicable to He^4 particles is the Bose-Einstein statistics.

(ii) The rare helium isotope, He^3, has one less neutron than He^4; therefore its number of substomic components is 5 which is an odd number. Conclusion: He^3 particles obey the statistics of antisymmtric particles, i.e., Fermi-Dirac statistics.

The markedly different properties of these very similar gases at low temperatures are explained and predicted by their respective statistical procedures.

(iii) All homonuclear, diatomic gases are composed of two identical atoms. The particle is the gas molecule.[16] The atoms come in pairs, e.g., O_2, N_2, etc. Thus their total numbers of subatomic parts are even numbers and these particles are Bosons, symmetric. They obey Bose-Einstein statistics.

(iv) Consider a heteronuclear diatomic gas. The hydrogen of hydrogen chloride, HCl, contributes 1 proton and 1 electron. The normal chlorine atom has 17 + 17 + 18. The total for HCl, 54, is even. Hydrogen chloride particles are symmetric—Bose-Einstein Statistics. Other heteronuclear diatomic gases may be antisymmetric.

Symmetry characteristics of particles are of practical significance to the studies of helium, hydrogen, electrons, and other particles as light or lighter than these, at practical temperatures and in principle to heavier particles at unattainably low temperatures.

Exercise 8. Classify each of the following particles as Bosons or Fermions. (a) diatomic hydrogen, H_2; (b) the hydrogen atom, H; (c) the hydrogen ion, H^+; (d) an electron, e^-; (e) a photon; (f) a phonon (photon and phonon are the names given to the massless particles of electromagnetic and acoustic radiation, respectively).

We are now able to classify indistinguishable particles as Bosons or Fermions. Further, we recognize that neither the counting procedures nor the combinatorial relationships illustrated for Boltzons (eqn. IIIA-7a) are applicable to these particles. Formulas for counting microstates of macrostates of indistinguishable particle systems have been derived (Ref. 5, Sect. 3.2). For Bosons and Fermions, respectively, they are

$$W_i^{BE} = \prod_j^J \frac{(g_j + n_{j,i} - 1)!}{(g_j-1)!\ n_{j,i}!} \qquad W_i^{FD} = \prod_j^J \frac{g_j!}{n_{j,i}!(g_j-n_{j,i})!} \qquad \text{(IIIA-7b,7c)}$$

Application of these formulas is illustrated in the following example and exercise. The macrostates of the 4 particle example, Table IIIA-2, will be used. (Macrostate

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[16]Recall that the term particle refers to the entire entity, not to its parts separately. Subsequent considerations (Unit IIIC) treating the components of gaseous molecules will depend on the evenness or oddness of portions of the total particle.