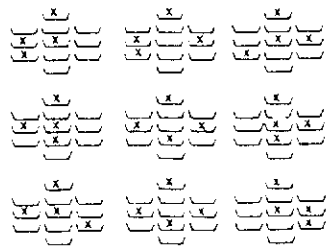
determinations depend on the number of particles and total energy but not on particle type.)

Example 8. Determine the number of microstates and the probability of each macrostate of the 4 particle system, Table IIIA-2, when the particles are Fermions.

Macrostate I. The distribution of Fermions in macrostate I is represented at the right. The Fermions are represented by indistinguishable X's and no more than one X is permitted in any state.[17] This gives a total of only 9 microstates.



Macrostate II. To determine this number of microstates we apply eqn. IIIA-7c

$$W_{II}^{FD} = \left(\frac{1!}{1!(1-1)!}\right)\cdot\left(\frac{3!}{0!(3-0)!}\right)\cdot\left(\frac{3!}{0!(3-0)!}\right)\cdot\left(\frac{3!}{3!(3-3)!}\right)\cdot\left(\frac{1!}{0!(1-0)!}\right) = 1$$

Macrostates III, IV, and V. Inspection of these distributions reveals that they require two particles in at least one state ($n_{3,III} = 4$, $g_3 = 3$; $n_{5,IV} = 2$, $g_5 = 1$; $n_{5,V} = 2$, $g_5 = 1$). Therefore, these macrostates have no microstates.[18,19]

$$W_{III}^{FD} = W_{IV}^{FD} = W_{V}^{FD} = 0$$

And the total number of microstates is

$$W^{FD} = \sum_{i=1}^{5} W_i^{FD} = 9 + 1 + 0 + 0 + 0 = 10$$

and the macrostate probabilities are

$$p_I^{FD} = 0.9; \qquad p_{II}^{FD} = 0.1; \qquad p_{III}^{FD} = p_{IV}^{FD} = p_V^{FD} = 0$$

Exercise 9. Determine the number of microstates and the probability of each of the macrostates for the 4 particle system, Table IIIA-2, with the particles treated as Bosons. (Answers in Table IIIA-3.)

Table IIIA-3 summarizes the probability results for the 4 particle example which has been repeated for Boltzons, Bosons, and Fermions.[20] Notice the pattern $W_i^{MB} > W_i^{BE} > W_i^{FD}$ We would expect Boltzons to have more microstates than Bosons since Boltzons are distinguishable. (New microstates occur due to interchanging of particles.) Bosons have more

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[17]A similar figure for Bosons would include additional microstates with the two particles on the 3rd level sharing each of the 3 states.

[18]Application of eqn. IIIA-7c to macrostates in which $g_j > n_{j,i}$ will call for negative factorials. Interpret such a condition as precluding the macrostate, $W_i = 0$.

[19]In some texts the macrostates of zero probability are not counted among the system macrostates.

[20]The macrostates appear in the table in the order of decreasing probability. This is not happenstance. They were arranged in that manner deliberately for our subsequent convenience. See property determinations below.