$$g_{pht}(\lambda) = \frac{8\pi V}{\lambda^4} \qquad n_{pht}(\lambda,T) = \frac{8\pi V}{\lambda^4}\frac{1}{[e^{(hc/\lambda kT)}-1]} \qquad \text{(IIID-3b,4b)}$$

The photon distribution allows us to establish the properties of a photon gas. For example, the energy distribution per unit volume, often called *Planck's Law* is given by[5,6]

$$u_{v\lambda}(T) = \frac{n_{pht}(\lambda,T)}{V}\frac{hc}{\lambda} = \frac{8\pi ch}{\lambda^5}\frac{1}{[e^{(hc/\lambda kT)}-1]} \qquad \text{(IIID-5)}$$

The form of this distribution is represented in Fig. IIID-1. Notice the pronounced maximum that separates the degenerate low energy (long wavelength) photons from the classical high energy (short wavelength) photons.[7] The position of the maximum is an important parameter for the classification of thermal radiation.[8]

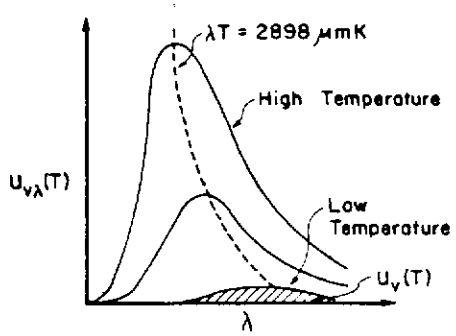

Fig. IIID-1. The Energy Distribution of Thermal Radiation

Example 1. Differentiate $u_{v\lambda}(T)$ with respect to wavelength and develop a condition for the position of its maximum.

The maximum is defined by

$$0 = \frac{du_{v\lambda}(T)}{d\lambda} = \frac{d}{d\lambda}\left\{\frac{8\pi hc}{\lambda^5[e^{(hc/\lambda kT)}-1]}\right\} = 8\pi hc \cdot \left\{\frac{-5}{\lambda^6[e^{(hc/\lambda kT)}-1]} + \frac{hc\, e^{(hc/\lambda kT)}}{\lambda^7 kT[e^{(hc/\lambda kT)}-1]^2}\right\}$$

$$= \frac{8\pi hc}{\lambda^6[e^{(hc/\lambda kT)}-1]^2}\left\{-5[e^{(hc/\lambda kT}-1] + \frac{hc}{\lambda kT}e^{(hc/\lambda kT)}\right\}$$

Thus the condition for the maximum has the form

$$(5 - x_{max})\, e^{x_{max}} = 5$$

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[5]Photon properties cannot be normalized on a per unit mass basis. Photon mass is zero. Molar normalization is possible but inconvenient as the number of photons is a variable.

[6]The wavelength distribution of radiant emission is also called Planck's Law. It differs only from eqn. IIID-5 by a constant, see Unit IIIE.

[7]As discussed above a system of photons never satisfies the Boltzmann limit. But within a system the high energy (short wavelength) tail of the distribution becomes classical, $e^{(hc/\lambda kT)} >> 1$. This portion of the distribution can be predicted by using classical methods of wave mechanics.

[8]Our development concerns radiation originating due to temperature alone. We exclude radiation that is mechanically generated, e.g., radio, TV, or other communication signals, lasers, etc.