The *turbine* work equals the compressor work in this idealized engine, $|w_c| = |w_t|$. Hence $T_4 = T_H - (T_2 - T_1) = 1700 - (488 - 232) = 1444$ K. The development of this work requires a pressure ratio equal to

$$r_{p,t} = \frac{p_4}{p_H} = \left(\frac{T_4}{T_H}\right)^{\gamma^*/(\gamma^*-1)} = \left(\frac{1444\ \text{K}}{1700\ \text{K}}\right)^{1.33/0.33} = 0.518 \quad \text{so} \quad p_4 = 4.94\ \text{atm} \cdot 0.518 = 2.56\ \text{atm}$$

The *nozzle* expands the gas to atmospheric pressure isentropically. The available pressure ratio is $r_{p,n} = p_L/p_4 = 0.185/2.56 = 0.0723$. The exhaust temperature is found from

$$T_5 = T_4 (r_{p,n})^{(\gamma^*-1)/\gamma^*} = 1444\ \text{K} \cdot (0.0723)^{0.33/1.33} = 752\ \text{K}$$

The exit velocity is determined from the steady state energy equation for the nozzle.

$$0 = c_p^*(T_4 - T_5) + \cancel{\frac{\hat{V}_4^2}{2}} - \frac{\hat{V}_e^2}{2} + \cancel{q_n} + \cancel{w_s} \quad \text{or} \quad \hat{V}_e = \sqrt{2c_p^*(T_4 - T_5)}$$

$$\hat{V}_e = \left[\frac{2 \cdot 1.33}{0.33} \cdot \left(\frac{8.314}{28.97}\right) \text{J/(g K)} \cdot (1444 - 752)\text{K} \cdot 10^3 \text{g m}^2/(\text{J s}^2)\right]^{\frac{1}{2}} = 1265\ \text{m/s}$$

Finally the net output from the engine is

$$\Delta ke = \frac{(\hat{V}_e^2 - \hat{V}_o^2)}{2} = \left[\frac{(1265)^2 - (201)^2}{2}\right] \text{m}^2/\text{s}^2 \cdot 10^{-3} \text{J s}^2/(\text{g m}^2) = 780\ \text{J/g}$$

Thus the thermal efficiency, Carnot efficiency, and efficiency ratio are

$$\eta_{th} = \frac{\Delta ke}{q_H} = \frac{780\ \text{J/g}}{1402\ \text{J/g}} = 0.556 \qquad \eta_c = 1 - \frac{216\ \text{K}}{1700\ \text{K}} = 0.873 \qquad R_\eta = \frac{0.556}{0.873} = 0.637$$

The propulsive and overall efficiencies are

$$\eta_p = \frac{2|\hat{V}_o/\hat{V}_e|}{1 + |\hat{V}_o/\hat{V}_e|} = \frac{2 \cdot (201/1265)}{1 + (201/1265)} = 0.274 \qquad \eta = \eta_{th}\eta_p = 0.556 \cdot 0.274 = 0.152$$

The thrust per unit exit area is found from eqn. VD-2a

$$\frac{F_i}{A_e} = \rho_e \hat{V}_e (\hat{V}_e - \hat{V}_o) = \frac{p_e}{RT_e} \hat{V}_e (\hat{V}_e - \hat{V}_o) = \frac{0.185\ \text{atm} \cdot 1.013 \cdot 10^5 \text{N/(m}^2 \text{atm)}}{\left(\frac{8.314}{28.97}\right) \text{J/(g K)} \cdot 752\ \text{K}} \cdot 1265\ \text{m/s}$$

$$\cdot\ (1265 - 201)\text{m/s} \cdot 10^{-3}\ \text{N s}^2/(\text{g m}) = 1.17 \cdot 10^5\ \text{N/m}^2$$

Exercise 2. Exmp. 2 analyzed the steady state performance of a turbojet engine at 40,000 ft altitude. This engine had to develop its takeoff power without ram compression, $\hat{V}_o = 0$, and in atmospheric surroundings at STP. With the same governing parameters $r_{p,c} = 20$, $T_H = 1700$ K, determine the pressure ratio available at the nozzle, the jet velocity, the thrust per unit exit area, η_{th}, R_η, η_p, and the overall efficiency. Also determine the ratio of thrust at takeoff to that at altitude.

The performance of an ideal turbojet is different for cruising and takeoff conditions. More accurate predictions of performance are obtained by incorporation of an isentropic efficiency for the diffuser, compressor, turbine, and nozzle (see Unit IVC, Sect. IIA). This is especially necessary in the case of the nozzle. A well-designed nozzle can be constructed to closely approximate reversible performance for *design* conditions. However, a nozzle of fixed geometry can perform ideally only while operating at design conditions, that is, only over a restricted range of pressure ratios. To perform ideally at