UNIT IIID

THE PROPERTIES OF GASES COMPOSED OF LOW-MASS PARTICLES

Free Electrons and Electromagnetic and Acoustical Radiation

Unit IIIC investigated the properties of indistinguishable independent particles in the Boltzmann limit, $n_j << g_j$. It was found that satisfaction of the limit is facilitated by small particle density, n_v (= N/V), high temperature, and large particle mass.[1]

$$\left.\frac{n_j}{g_j}\right|_{max} \sim n_v\left(\frac{h^2}{2\pi m^* kT}\right)^{3/2} \overset{?}{<<} 1 \qquad \text{(IIIC-15)}$$

In the consideration of molecular gases we found that only the lightest molecules, helium and hydrogen, ever fail to satisfy the limit, see Exer. 7, Unit IIIC. Thus, in discussing systems that do not normally satisfy the limit we need consider only particles whose mass is less than that of a proton.

Exercise 1. Each atom of crystalline copper contributes a single electron to its free electron gas. Determine the magnitude of $n_v(h^2/2\pi m^* kT)^{3/2}$ for the free electron gas in solid copper at normal temperatures. Also, determine the hypothetical temperature at which the limit would be satisfied by setting it equal to 10^{-2}.

Exer. 1 shows that the free electron gas within a metallic lattice would never satisfy the Boltzmann limit. Systems of this type are said to be *degenerate*. Since a single electron constitutes an odd number of subatomic components, electricity is conducted by *degenerate Fermions*.[2] The only common particles lighter than an electron are massless. And the wave-particle duality principle allows us to envision electromagnetic radiation either as waves or as a gas of massless particles translating at the speed of light, *photons*. Similarly, we may envision acoustical radiation either as waves or as a gas of massless particles translating at the speed of sound, *phonons*. Since zero is an even number of subatomic components, photons and phonons are Bosons. While we cannot apply the limit test directly to these massless particles, we would not expect them to satisfy the Boltzmann limit. Photons and phonons are *degenerate Bosons*. Thus we must employ the exact distribution function to analyze free electrons and waves.

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[1]This expression, which involves volume per particle rather than pressure, is equivalent to eqn. IIIC-15, see Appendix IIIC-A, Exmp. A1.

[2]Pairs of electrons would be Bosons. The phenomenon of superconductivity occurs when electron wave functions are coupled creating stable electron pairs. These free electron pairs undergo a Bose degeneracy to establish the characteristics of the superconductor.