Since the $n_{j,i}$ are linearly independent, the satisfaction of eqn. IIIB-7 requires that the bracketed term be zero for each j.[10] Thus, the macrostate that satisfies eqns. IIIB-6, the most probable (i = mp), is determined.

$$\ln\left(\frac{g_j}{n_{j,mp}}\right) = \alpha + \beta\varepsilon_j(V) \qquad n^{MB}_{j,mp} = \frac{g_j}{e^{\alpha+\beta\varepsilon_j(V)}} \tag{IIIB-8}$$

This result is not yet useful since α and β are unknown. For Maxwell-Boltzmann statistics α can be readily determined as a function of N, β, g_j and $\varepsilon_j(V)$. Substitution of the expression for $n^{MB}_{j,mp}$ into eqn. IIIB-6b $N = \left(\sum_j n_{j,i}\right)$ yields[11]

$$\alpha = \ln\left(\frac{Z}{N}\right) \quad \text{or} \quad e^{-\alpha} = \frac{N}{Z}$$

The function Z is called the *partition function.*

$$Z(V,\beta) \equiv \sum_j g_j\, e^{-\beta\varepsilon_j(V)}$$

The partition function is a sum that appears frequently in calculations of thermodynamic properties. Its definition saves considerable writing.

To determine β we first write entropy as a function of β,U, the partition function, $Z(\beta,V)$, and N. For a Maxwell-Boltzmann system the expression for entropy of a macrostate (as shown in Appendix IIIB-A) is

$$S^{MB}_i = k \ln W^{MB}_i = k\left\{\left[\sum_j n^{MB}_{j,i} \ln\left(\frac{g_j}{n^{MB}_{j,i}}\right)\right] + N \ln N\right\} \tag{IIIB-A3}$$

To find the equilibrium entropy we substitute the equilibrium distribution into the above equation as follows

$$n^{MB}_{j,mp} = \frac{N g_j\, e^{-\beta\varepsilon_j(V)}}{Z} \quad \text{and} \quad \ln\left(\frac{g_j}{n^{MB}_{j,mp}}\right) = \ln Z - \ln N + \beta\varepsilon_j(V)$$

Therefore

$$S^{MB}_{eq} = k\left\{\left[\sum_j n_{j,i}(\ln Z - \ln N + \beta\varepsilon_j)\right] + N \ln N\right\}$$
$$= k\left(\ln Z \sum_j n_{j,i} - \ln N \sum_j n_{j,i} + \beta \sum_j n_{j,i}\,\varepsilon_j + N \ln N\right)$$

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[10] This statement is an oversimplification as there is some interdependence through the constraints. A more rigorous treatment yields the same conclusions.

[11] The steps are $$N = \sum_j n^{MB}_{j,i} = \frac{\sum_j g_j\, e^{-\beta\varepsilon_j(V)}}{e^{\alpha}}$$