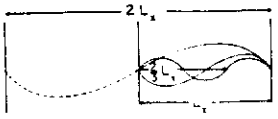
A. The Properties of the Photon Gas

Quantum mechanics was developed by Max Planck as a necessary step to accurately predict the properties of electromagnetic radiation. Planck hypothesized that radiation of frequency ν must possess energy in multiples of a minimum quantum, $h\nu$. Thus, the distribution of a gas of photons in equilibrium at temperature T, using eqn. IIID-1, is

$$n_j^o = \frac{g_j}{e^{h\nu_j/kT} - 1}$$

In order to apply this distribution to determine the properties of a photon gas, we require the sequence of allowed frequencies, ν_j, and their corresponding degeneracies, g_j. The allowed frequencies are governed by the geometry of the enclosure, specifically by its *boundary conditions*. The amplitude of waves within an enclosure must vanish at its boundaries. For example, the waves in a one-dimensional enclosure, as shown, are limited to

$$\lambda_\ell = \frac{2L_x}{\ell} \qquad \ell = 1, 2, 3, \ldots$$



The relationship between frequency and wavelength is $\nu \equiv c/\lambda$.

In a three-dimensional cubic enclosure the combined three-dimensional wavelength limits give the following allowed frequencies

$$\nu_{\ell,m,n} = c\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}\right)^{\frac{1}{2}} = \frac{c}{2V^{1/3}}\left(\ell^2 + m^2 + n^2\right)^{\frac{1}{2}} \qquad n, m, \ell = 1, 2, 3, \ldots$$

Note that this expression for the *discrete frequencies* has the form of eqn. IIIC-12. It is expressed as the product of a physical parameter and a dimensionless function of the sequence of integers (n, m, ℓ). Thus, the characteristic temperature that prescribes the frequency spacing of a photon gas is

$$\theta_{pht} = \frac{h\Delta\nu}{k} = \frac{hc}{2kV^{1/3}} \qquad \text{(IIID-2)}$$

Ans. 1. The volume of a free electron is the same as that of an atom. Using the data of Tables IA-1, 2 and ID-2

$$n_{v,el} = \frac{\rho_{Cu} n_{e,Cu} N_A}{M_{Cu}} = \frac{8.84\ \text{g/cm}^3 \cdot 1\ \text{elect/atom} \cdot 6.023 \cdot 10^{23}\ \text{atoms/(g mol)}}{63.43\ \text{g/(g mol)}} = 8.47 \cdot 10^{22}\ \text{elect/cm}^3$$

and

$$n_{v,el}\left(\frac{h^2}{2\pi m^* kT}\right)^{3/2} = 8.47 \cdot 10^{22}\ \text{elect/cm}^3$$

$$\left[\frac{(6.625 \cdot 10^{-27}\ \text{erg s/elect})^2}{2\pi \cdot 9.109 \cdot 10^{-28}\ \text{g/elect} \cdot 1.38 \cdot 10^{-16}\ \text{erg/(elect K)} \cdot 300\ \text{K}}\right]^{3/2} = 6.75 \cdot 10^3\ \text{elect/state}$$

Obviously the limit is not satisfied at ordinary temperatures. It would theoretically be satisfied ($n_j/g_j|_{max} \sim 10^{-2}$) if the temperature were increased to $2.31 \cdot 10^6$K.