The Fermi function, $F(\varepsilon)$, is sketched in Fig. IIID-4 for different temperature ranges. (Remember that no more than one Fermion can occupy the same state, $0 \le F(\varepsilon) \le 1$.) At absolute zero temperature, $F(\varepsilon)$ is a step function—all of the states up to the Fermi level, $\varepsilon_F^\circ = \varepsilon_F(0\ \mathrm{K})$, are full and all of the states above this level are empty. Notice that the distribution corresponding to $T \ll \varepsilon_F^\circ/k$, the degenerate range, departs only slightly from the $T = 0$ K distribution. It will be seen below that all temperatures below the melting point of metals lie in the range $T \ll \varepsilon_F^\circ/k$. Hence the distribution at 0 K, or the value of ε_F°, is extremely significant in determining the properties of free electrons.

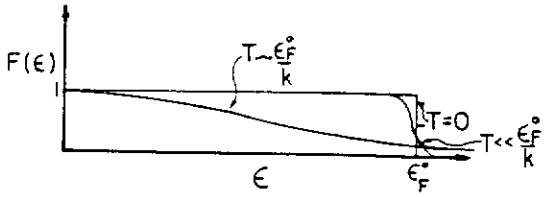

Fig. IIID-4. The Fermi Function

The expression for the degeneracy in conjunction with the simplified distribution function at zero temperature $[n_{el}(\varepsilon) = g_{el}(\varepsilon)$ at $\varepsilon \le \varepsilon_F^\circ$; $n_{el}(\varepsilon) = 0$ at $\varepsilon > \varepsilon_F^\circ]$ and the conservation of particles condition, $N_{el} = \int_0^\infty dn_{el}$, allows an expression for ε_F° to be developed.

Example 8. Derive an expression for ε_F° in terms of the valence, molecular weight, and density of the crystal by applying the conservation of particles condition at absolute zero.

At zero temperature the distribution function is given as[22]

$$n_{el}\Big|_{T=0} = g_{el}(\varepsilon) = 4\pi V\left(\frac{2m_{el}^*}{h^2}\right)^{3/2} \varepsilon^{\frac{1}{2}} \qquad \varepsilon \le \varepsilon_F^\circ \qquad\qquad n_{el}\Big|_{T=0} = 0 \qquad \varepsilon > \varepsilon_F^\circ$$

Thus we can express the total number of electrons as

$$N_{el} = \int_0^\infty g_{el}(\varepsilon)d\varepsilon = 4\pi V\left(\frac{2m^*}{h^2}\right)^{3/2} \int_0^{\varepsilon_F^\circ} \varepsilon^{\frac{1}{2}}d\varepsilon + \int_{\varepsilon_F^\circ}^\infty (0)d\varepsilon$$

$$N_{el} = 4\pi V\left(\frac{2m_{el}^*}{h^2}\right)^{3/2} \cdot \frac{2}{3}\,\varepsilon_F^{\circ\ 3/2} \qquad \text{or} \qquad \varepsilon_F^\circ = \frac{h^2}{8m_{el}^*}\left(\frac{3\,N_{el}}{\pi V}\right)^{2/3}$$

[22]The statement "at zero temperature" should not be taken too literally. At zero temperature, the series cannot be represented as a continuous function and eqn. IIID-18 is invalid.

Ans. 15. Substituting $m_{el}^* = 9.109 \quad 10^{-31}$kg

$$V = \left(\frac{h^2}{8\,m_{el}^*k\theta_{tr}^\circ}\right)^{3/2} = \left[\frac{(6.625 \cdot 10^{-34}\,\mathrm{Js/elect})^2}{8 \cdot 9.109 \cdot 10^{-31}\,\mathrm{kg} \cdot 1.380 \cdot 10^{-23}\,\mathrm{J/(elect\ K)}\,1\,\mathrm{K}}\right]^{\frac{3}{2}} = 2.88 \cdot 10^{-22}\,\mathrm{m}^3$$

or a cube 661 Å on a side.