The transformation of this distribution to velocity must satisfy the conservation of probability condition

$$1 = \int_0^\infty \int_0^\infty \int_0^\infty \rho_{tr}(\ell,m,n)\, d\ell\, dm\, dn = \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \rho_{tr}(V_x,V_y,V_z)\, dV_x dV_y dV_z$$

Then substituting for ℓ in terms of V_x

$$\left(\frac{h^2}{8m^* V^{2/3}}\right)\ell^2 = \frac{m^*}{2} V_x^2 \quad \text{or} \quad \ell = \pm\left[\frac{4(m^*)^2 V^{2/3}}{h^2}\right]^{\frac{1}{2}} V_x \qquad d\ell = \pm\left[\frac{4(m^*)^2 V^{2/3}}{h^2}\right]^{\frac{1}{2}} dV_x$$

$$\text{and} \quad \ell = 0, \quad V_x = 0; \quad \ell \Rightarrow \infty, \quad V_x \Rightarrow \pm\infty.$$

Similarly for m and n in terms of V_y and V_z, we have

$$1 = \frac{h^3}{V(2\pi m^* kT)^{3/2}} \int_0^\infty \int_0^\infty \int_0^\infty e^{\frac{-m^*}{2kT}\left(V_x^2+V_y^2+V_z^2\right)} \left|\frac{4(m^*)^2 V^{2/3}}{h^2}\right|^{3/2} dV_x\, dV_y\, dV_z$$

$$= \frac{1}{2^3}\left(\frac{2m^*}{\pi kT}\right)^{3/2} \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty e^{\frac{-m^*}{2kT}\left(V_x^2+V_y^2+V_z^2\right)} dV_x\, dV_y\, dV_z$$

where the factor $1/2^3$ comes from extending all three limits from zero to $-\infty$. The integrand of this probability integral is the velocity distribution, $\rho(\bar{V})$. It can be expressed in the simpler form of eqn. IIIE-1 by substituting $R = \bar{R}/M = N_A k/N_A m^*$.

In a stationary gas the probability that a particle will have the velocity $\bar{V}$ is

$$\rho(\bar{V}) = \frac{e^{-\bar{V}^2/2RT}}{(2\pi RT)^{3/2}} \qquad \text{(IIIE-1)}$$

A sketch of the distribution in two of its three directions is shown in Fig. IIIE-1. Notice that the most probable velocity is zero; $\rho_{max} = \rho(\bar{V} = 0) = (2\pi RT)^{-3/2}$ The velocity distribution is symmetric with respect to the origin. Thus, the average of any odd function of velocity over the entire velocity space is zero. This supports our expectation that there is no net transport within a stationary gas in equilibrium.[6] In that respect it is useful to ignore the direction of individual particle motions and consider the *speed distribution* $V \equiv |\bar{V}|$

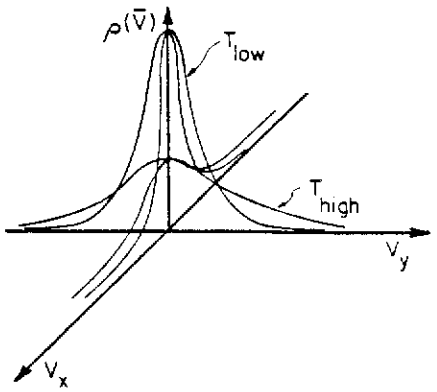

Fig. IIIE-1. Velocity Distribution in a Molecular Gas

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[6]The distribution in an equilibrium gas that is moving with velocity $\bar{V}_o$ is readily shown to be

$$\rho(\bar{V}) = \frac{e^{-(\bar{V}-\bar{V}_o)^2/2RT}}{(2\pi RT)^{3/2}}$$

This distribution is symmetric with respect to $\bar{V}_o$. And there is still no molecular transport except that contributed by the mean particle motion, viz., convection.