Thus necessary and sufficient conditions for a reversible heat effect are that it occur without a temperature differential.

The first term in eqn. IIIB-26 changes its sign but not its magnitude, if the direction of heat flow is reversed. It can be thought of as a reversible transfer of entropy between the system and its surroundings.

$$dS_{th,rev} = \frac{đQ}{T_B} \qquad \text{(IIIB-28)}$$

When this expression is solved for $đQ$ an analogy with intrinsic work is recognized

$$đQ = T_B dS_{th,rev} \qquad đW_k^i = \bar{F}_{B,k} \cdot d\bar{X}_k \quad .$$

In this analogy boundary temperature, T_B, plays the role of boundary force, $\bar{F}_{B,k}$, and the reversible thermal entropy change, $dS_{th,rev}$, plays the role of displacement, $d\bar{X}_k$.[17] The analogy can be extended to recognize the reversible and irreversible components of a heat interaction. This is accomplished by substituting eqn. IIIB-25 for the boundary temperature.

$$đQ = T_{eq} dS_{th,rev} + \delta T_{neq} dS_{th,rev}$$

$$đQ_{rev} = T_{eq} dS_{th,rev} \qquad đQ_{irr} = \delta T_{neq} dS_{th,rev} \qquad \text{(IIIB-29a,b)}$$

If this analogy is to be complete we require that $đQ_{irr} \geq 0$. We leave this verification to the following exercise.

Exercise 10. Verify that $đQ_{irr}$ is positive semidefinite. Use a physical argument based on the relationship between the signs of δT_{neq} and $dS_{th,rev}$.

Discussion of the analogy is continued by examination of the relationship between thermal entropy production and irreversible heat.

Example 7. Show that thermal production is analogous to mechanical production by demonstrating that $dS_{th,irr} = đQ_{irr}/T_{eq}$ just as $dS_m = đW_{irr}/T_{eq}$.

The proof can be completed with eqn. IIIB-27 by substitution of $dS_{th,irr} = (đQ\delta T_{neq})/(T_B T_{eq})$ and the introduction of eqn. IIIB-28, $dS_{th,irr} = dS_{th,rev}(\delta T_{neq}/T_{eq})$. Finally the substitution of eqn. IIIB-29b gives

$$dS_{th,irr} = \frac{đQ_{irr}}{T_{eq}}$$

We have now codified the mechanisms of entropy change within a closed system into three components

[17] There is an apparent discrepancy in this analogy in that $dS_{rev,th}$ is only part of the entropy change, whereas $d\bar{X}$ is all of the mechanical displacement. The rationale is that only that part of the entropy that is conserved can be considered to cross the boundary. All of the displacement is conserved.