Thus, $$n_2 = \frac{Ng_2 e^{-\varepsilon_2/kT}}{Z} = \frac{10^{18} \cdot 3.2 \cdot 10^{21} \cdot \exp(-27.1)}{6.4 \cdot 10^{22}} = 8.5 \cdot 10^4 \text{ particles}$$

$n_1 = N - n_2 \approx 10^{18}$ particles $U = n_1\varepsilon_1 + n_2\varepsilon_2 = 10^{18} \cdot 0 + 8.50 \cdot 10^4$ part $\cdot\ 2.88 \cdot 10^{-20}$ J/part $= 2.45 \cdot 10^{-15}$ J.

(b) After pumping, $n_2 = 5.00 \cdot 10^{16}$, and $n_1 = N - n_2 = 95.0 \cdot 10^{16}$. We solve for the new temperature from

$$\frac{n_2}{n_1} = \frac{g_2 e^{-\varepsilon_2/kT}}{g_1 e^{-\varepsilon_1/kT}} = \frac{g_2}{g_1} e^{-(\varepsilon_2-\varepsilon_1)/kT}$$

Or,

$$T = \frac{-(\varepsilon_2-\varepsilon_1)}{k \ln\left(\frac{n_2/g_2}{n_1/g_1}\right)} = \frac{-2.88 \cdot 10^{-20} \text{ J/part}}{1.38 \cdot 10^{-23} \text{ J/part} \cdot \ln\left(\frac{5 \cdot 10^{16} \cdot 6.4 \cdot 10^{22}}{95 \cdot 10^{16} \cdot 3.2 \cdot 10^{21}}\right)} = -4.07 \cdot 10^4 \text{ K}$$

A negative temperature? Yes, systems with a limited number of allowed (bounded) states can and do achieve negative temperatures. The positive temperatures of our experience are defined on the basis of equilibrium distributions. As seen from the expression for T above, the nonequilibrium condition necessary for laser operation, $(n_2/g_2) > (n_1/g_1)$, is responsible for its negative temperature. But, negative temperatures are high, not low, temperatures, as can be seen from this example— the conditions that led to a negative temperature resulted from energy addition to the system.[14] The new energy is

$$U = \sum_j n_j\varepsilon_j = 0 + 5.00 \cdot 10^{16} \text{ part} \cdot 2.88 \cdot 10^{-20} \text{ J/part} = 1.44 \cdot 10^{-3} \text{ J}$$

Once the system has been pumped as described, a discharge is triggered with a signal at the proper frequency. As the electrons drop back into lower energy states energy is released in the form of coherent emission of photons from different positions in the crystal. This accumulated effect causes the emission of a very high intensity monochromatic beam whose wavelength is given by

$$\lambda = \frac{hc}{\varepsilon_2-\varepsilon_1} = \frac{6.626 \cdot 10^{-34} \text{ J} \cdot \text{s/part} \cdot 2.998 \cdot 10^8 \text{ m/s}}{2.88 \cdot 10^{-20} \text{ J/part}} = 6897 \text{ Å}. \quad \text{A vivid red light.}$$

Exercise 2. An isolated Maxwell-Boltzmann system has three energy levels ($\varepsilon_1 = 0$, $g_1 = 10^4$; $\varepsilon_2 = 2.76 \cdot 10^{-21}$ J/part, $g_2 = 3.00 \cdot 10^4$; $\varepsilon_3 = 4.14 \cdot 10^{-21}$ J/part, $g_3 = 3.00 \cdot 10^4$). The system is found in a macrostate with $n_1 = 6430$, $n_2 = 2610$, $n_3 = 960$. (a) What are the overall constraints on this system, N and U? (b) Determine whether the specified (continued)

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[14]Many students may find it discomforting that negative temperatures are "hotter" than normal positive temperatures. That is, that temperature is not a monotonically increasing function. (There is a discontinuity as the scale goes from $+\infty$ to the next higher temperature, $-\infty$. The highest temperature is -0.) This characteristic can be altered by defining temperature as minus β, viz., $\theta \equiv -\beta$. A correspondence between the two scales T and θ in the order of increasing "hotness" is ($T = +0$, $\theta = -\infty$; $T = +\infty$, $\theta = -0$; $T = -\infty$, $\theta = +0$; $T = -0$, $\theta = +\infty$).