example showed that in the indistinguishable limit the number of microstates state is the same for Fermions and Bosons. But it is smaller than that for a factor of N![1]

recognized that the distribution of particles over the levels is the same for statistics in the Boltzmann limit, we can borrow the expression for α obtained ns in Unit IIIB, $e^{\alpha} = Z(T,V)/N$, where $Z = \sum_{j=1} g_j \exp[-\varepsilon_j(V)/kT]$. This gives brium distribution and equivalently the probability that a particle will be in vel in the limit $n_j \ll g_j$.

$$n_j^{il} = n_j^{MB} = \frac{Ng_j e^{-\varepsilon_j(V)/kT}}{Z(T,V)} \qquad \rho_j^{il} = \rho_j^{MB} = \frac{n_j}{N} = \frac{g_j e^{-\varepsilon_j(V)/kT}}{Z(T,V)} \qquad \text{(IIIC-1a,b)}$$

e distribution functions become the same in the limit, the entropy of indistin- Bosons and Fermions remains distinct from that of distinguishable Boltzons. The a macrostate was defined by eqn. IIIB-1 ($S_i \equiv k\ln W_i$). This is applicable to statistics. Thus, using the result of Exmp. 1 we have

$$S^{MB} = k\ln W^{MB} \qquad S^{il} = k\ln\left(\frac{W_{MB}}{N!}\right) = S^{MB} - k\ln N!$$

tibrium entropy of a system of Boltzons was formulated in Unit IIIB as

$$S^{MB} = \frac{U}{T} + Nk\ln[Z(T,V)] \qquad \text{(IIIB-13)}$$

substitute this result and Stirling's approximation ($\ln N! = N\ln N - N$, see Appendix -A), we obtain

$$S^{il} = \frac{U}{T} + Nk\ln\left[\frac{Z(T,V)}{N}\right] + Nk$$

e relationships provide the basis for the determination of the properties of Boltzons, those of Fermions and Bosons in the indistinguishable limit. Microscopic property rmination capitalizes on the similarity between substances. It is therefore convenit to formulate properties in *molar units*. We do so by substituting $nN_A = N$ and $n\bar{v} = V$. gives (note that $N_A k = \bar{R}$)

$$\bar{n}_j = \frac{n_j}{n} = \frac{N_A g_j e^{-\varepsilon_j(n\bar{v})/kT}}{Z(T,n\bar{v})} \qquad \text{(IIIC-1a)}$$

$$\bar{s}^{MB} = \frac{\bar{u}}{T} + \bar{R}\ln[Z(T,n\bar{v})] \qquad \bar{s}^{il} = \frac{\bar{u}}{T} + \bar{R}\ln\left[\frac{Z(T,n\bar{v})}{nN_A}\right] + \bar{R} \qquad \text{(IIIC-2a,b)}$$

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[1] N! is the number of permutations, or distinct rearrangements, of N items. This is ften advanced as a physical interpretation of the appearance of that factor. However it s not an acceptable argument as the $n_j!$ which appears in the denominator of all three statistical formulae also has its origin in the distinguishability characteristic. It ppears in the Boltzon expression, so that the order in which the distinguishable particles are placed in the levels is not counted. And it appears in the Bose and Fermi expressions to discount the counting of arrangements of the indistinguishable particles among the states in the levels. (See Ref. 4, pg 140.)