The solution of the equation with constraints is $n_{j,mp}$. Unit IIIA presented expressions for W_i for the independent particle systems for the three statistics. However, before substitution of W_i explicitly we can perform some operations symbolically.

There are two procedures to solve the extremum problem prescribed by eqns. IIIB-6.[8] These are the method of Lagrange Multipliers and the method of Steepest Descent. We will use the method of *Lagrange Multipliers*. To use this method we multiply eqns. IIIB-6b and c by unknown parameters α and β, respectively, then subtract them from eqn. IIIB-6a.[9]

$$d\left(\frac{S}{k} - \alpha N - \beta U\right) = d[\ell n\ W(n_{j,i})] - \alpha \sum_j \delta n_{j,i} - \beta \sum_j \varepsilon_j(V)\delta n_{j,i} = 0 \qquad \text{(IIIB-7)}$$

Eqn. IIIB-7 can be solved to obtain the most probable distribution as a function of the unknown multipliers, $n_{j,mp}(\alpha,\beta)$. Then the solution can be completed by identifying the physical meaning of α and β by further consideration of the physical conditions on U, N, and S. To proceed we need an explicit expression for W_i. These expressions were presented in Unit IIIA for the three statistics. We will first treat Boltzons.

Note to the Student. The results developed below are important though algebraically tedious. The logic for some of the algebraic steps and approximations lies in knowing the forms of results desired. Do not let the algebraic detail obscure its intent or results. Study or reproduce the algebra (most of which is presented in the appendices) only so far as you personally require to believe and understand the results.

A. The Distribution Function for Boltzons

In Unit IIIA the number of microstates (W_i) of a macrostate (i) of a Maxwell-Boltzmann system was shown to be calculable using eqn. IIIA-7a, $W_i^{MB} = N! \prod_j (g_j^{n_{j,i}})/(n_{j,i}!)$. The derivative of $\ell n\ W_i^{MB}$ is derived in Appendix IIIB-A. The result is

$$d(\ell n\ W_i^{MB}) = \sum_j \ell n \left(\frac{g_j}{n_{j,i}}\right) \delta n_{j,i}$$

Substituting this into eqn. IIIB-7 we obtain

$$\sum_j \left[\ell n \left(\frac{g_j}{n_{j,i}}\right) - \alpha - \beta\varepsilon_j(V)\right] \delta n_{j,i} = 0$$

[8]The term extremum refers to maximum and minimum, as does the condition dS = 0. The second derivative of S will show that our results pertain to the maximum.

[9]Lagrange's method is explained in Ref. 3, Appendix A.

Ans. 1. No occupied macrostate can have less than one microstate, $W_i \geq 1$. Since $\ell n\ 1 = 0$, the lowest possible entropy is zero, $S_{min} = 0$. Such a state would be completely certain, i.e., totally ordered. This answer supports the disorder analogy.