pattern that relates macrostate distributions to system properties. Once this pattern is understood, we will be able to avoid the tedium of trial-and-error procedures.

Since macrostate distributions cannot be found analytically, our examination of procedure and results, and the discovery of trends must be accomplished in terms of examples.[12] For consistency and simplicity, we will use the energy levels and degeneracies developed for the particle in a cubic box. The energy levels of this system were written in terms of the constant, B_o $[(\equiv h^2/(8m^*V^{2/3})]$, and the quantum numbers ℓ,m, and n = 1,2,3,..., and all possible combinations, as

$$\varepsilon_j = \varepsilon_{\ell,m,n} = B_o(\ell^2 + m^2 + n^2) \qquad \text{(IIIA-5)}$$

or specifically ε_j/B_o =.3,6,9,11,12,14,17,18,19,21,22,24,... (see Table IIIA-1). In this example the internal energy of the system must be an integral multiple of B_o, $U = L\ B_o$. And, since the minimum energy of any single particle is 3 B_o, the minimum total energy is $U_{min} = 3\ NB_o$. Thus $L \geq 3\ N$. Subject to these constraints, we propose to draw physical conclusions by considering successively larger numbers of point particles isolated in a cubic box. As a smallest example consider the following.

Example 4. Determine the macrostate distributions of a system of three particles with a total energy of 27 B_o isolated within a cubic box.

This problem will require trial and error to determine all distinct sets of $n_{j,i}$ that satisfy the constraints, eqns. IIIA-6.

$$N = 3 = \sum_{j=1} n_{j,i} \qquad \text{and} \qquad U = 27\ B_o = \sum_{j=1} n_{j,i}\varepsilon_j$$

To begin, we can divide the 27 B_o energy evenly among the three particles. Thus each particle would have 9 B_o and Table IIIA-1 shows this to be acceptable. Thus macrostate I is

$$\text{I.}\quad n_{1,I} = n_{2,I} = 0,\quad n_{3,I} = 3,\quad n_{4,I} = n_{5,I} = \ldots = n_{J,I} = 0$$

$$\text{Check:}\quad 0\cdot 3\ B_o + 0\cdot 6\ B_o + 3\cdot 9\ B_o + 0\cdot 11\ B_o + 0\cdot 12\ B_o = 27\ B_o$$

We have established a first macrostate and that the levels above or below level 3 can possess at most two particles for any macrostate. If level four has two particles their energy is 22 B_o and the third particle must have 5 B_o. This is not possible; 5 B_o is not an allowed energy. Trying two particles in level five we find that the third particle in level one satisfies the constraint equations.

$$\text{II.}\quad n_{1,II} = 1,\quad n_{2,II} = n_{3,II} = n_{4,II} = 0,\quad n_{5,II} = 2,\quad n_{6,II} = \ldots = 0$$

$$\text{Check:}\quad 1\cdot 3\ B_o + 0\cdot 6\ B_o + \ldots + 2\cdot 12\ B_o + \ldots = 27\ B_o$$

Two particles cannot occupy levels above level five. Now we try two particles in level two. This requires the third to have 15 B_o which is not allowed. However, two particles in level one and one in level ten constitute a macrostate.

[12]Whereas examples are generally intended to be supplements to the text, in Unit IIIA important concepts are established by means of example.