The characteristics of Fermi distribution were delineated in Fig. IIID-4, for the various temperature ranges defined in terms of ε_F°/k. With the expression for ε_F° developed in Exmp. 8, we can quantify the comparison of temperatures by defining a characteristic *Fermi temperature.*

$$\theta_F = \frac{\varepsilon_F^\circ}{k} = \frac{h^2}{8m_{el}^* k}\left(\frac{3n_e N_A}{\pi \bar{v}}\right)^{2/3} \qquad \text{(IIID-20)}$$

where n_e is the valence of the atom and $\bar{v}$ is the molar crystalline specific volume.

Exercise 16. Determine the magnitude of the Fermi temperature for aluminum, copper, and lead. (The necessary valences, molecular weights, and densities are found in Tables IA-1 and ID-2.)

The answer to Exer. 16 and Table IIID-1 demonstrate that the values of θ_F are so large that the free electrons of a metallic crystal are always within the degenerate limit, $T \ll \theta_F$. Their distribution is the rounded step of Fig. IIID-4, departing only slightly from the T = 0 K distribution. The average energy of the electrons can be estimated from the absolute zero distribution.

Example 9. Determine the minimum average particle energy for a free electron gas, i.e., at absolute zero temperature. Compare this energy to that which the electrons would have if they were classical translating particles, $\bar{\varepsilon}_{tr,class} = (3/2)\,kT$, by defining the *kinetic temperature* of free electrons: $T_{kin} \equiv (2/3)\bar{\varepsilon}_{tr}/k$.

The average energy of a system of particles is defined by $\bar{\varepsilon} \equiv \sum_j \rho_j \varepsilon_j$. For a system of free electrons this general relationship becomes

$$\bar{\varepsilon}_{el} = \frac{1}{N_{el}} \int_0^\infty \varepsilon n_{el} d\varepsilon$$

At absolute zero, $n_{el} = g_{el}$ for $\varepsilon < \varepsilon_F^\circ$ and $n_{el} = 0$, $\varepsilon > \varepsilon_F^\circ$.

$$\bar{\varepsilon}_{el}^\circ = \frac{1}{N_{el}} \int_0^\infty \varepsilon g_{el}(\varepsilon) d\varepsilon = \frac{4\pi V}{N_{el}}\left(\frac{2m_{el}^*}{h^2}\right)^{3/2} \int_0^{\varepsilon_F^\circ} \varepsilon\, \varepsilon^{\frac{1}{2}} d\varepsilon = \frac{4\pi V}{N_{el}}\left(\frac{2m_{el}^*}{h^2}\right)^{3/2} \left(\frac{2}{5}\,\varepsilon_F^{\circ 5/2}\right)$$

Then substituting from Exmp. 8 for $N_{el} = 4\pi V(2m^*/h^2)^{3/2}(2/3)\cdot\varepsilon_F^{\circ 3/2}$, we obtain $\varepsilon_{el}^\circ = \frac{3}{5}\,\varepsilon_F^\circ$.

Employing the definition of the kinetic temperature, we find

$$T_{kin}^\circ = \frac{2}{3}\,\frac{\bar{\varepsilon}_{el}^\circ}{k} = \frac{2}{3}\cdot\frac{3}{5}\,\frac{\varepsilon_F^\circ}{k} = \frac{2}{5}\,\theta_F$$

Thus the kinetic temperature of free electrons even in the limit of zero absolute is enormous. The average electron still moves very rapidly.[23]

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[23]The concept of kinetic temperature provides a means to compare the energy of degenerate free electrons to a hypothetical classical energy in the more recognizable units of temperature. The thermodynamic temperature of the electrons remains that of the lattice with which they are in equilibrium.