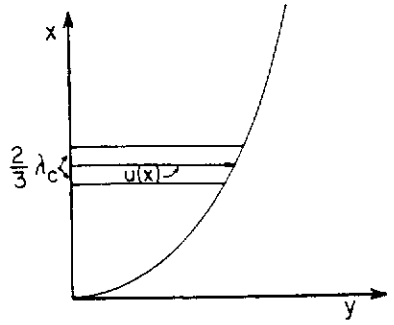
$$j''_{\Gamma} = -\frac{4\dot{n}''}{3}\lambda_c \frac{\partial\hat{\Gamma}(x)}{x} \qquad \text{(IIIE-18)}$$

We can apply this result to momentum, heat, and species transport.



Momentum Transport. Consider fluid flow parallel to a surface as shown. The particles in contact with the wall are stationary. As a result a velocity defect zone is developed in the vicinity of the surface. The thickness and velocity distribution within this "*boundary layer*" are governed by momentum transport between the adjacent layers. The average y momentum of the particles at distance x from the wall is $\hat{\Gamma}(x) = m^* u(x)$. The momentum flux is predicted by eqn. IIIE-18 and it equals the negative of the shear stress at that point.[15]

$$j''_{m^*u} = -\frac{4}{3}\dot{n}''\lambda_c \frac{\partial m^* u(x)}{\partial x} = -\sigma_{xy}$$

The proportionality between the velocity gradient and shear stress is normally reported in terms of the constitutive equation, $\sigma_{xy} = \mu\partial u/\partial x$, that defines the Newtonian viscosity of a fluid, μ. When we substitute this definition into the previous expression for the momentum flux, we obtain a general expression for the *viscosity* of a fluid

$$\mu = \frac{4}{3}\dot{n}''\, m^*\lambda_c = \frac{\rho\hat{V}\lambda_c}{3} \qquad \text{(IIIE-19a)}$$

A similar procedure can be used to develop a general expression for thermal conductivity.

Heat Transport. Continuum energy transport by means of any carrier particle, i, can be obtained by identifying the transported quantity as the average energy per carrier particle, $\Gamma_i(x) = \bar{u}_i/N_A$. Application of eqn. IIIE-18 gives the continuum heat flux contribution of each type of carrier particle[16]

$$j''_{th,i} = \frac{4}{3}\dot{n}''_i\lambda_{c,i}\frac{\partial}{\partial x}\left[\frac{\bar{u}_i[T(x)]}{N_A}\right] = -\frac{4}{3}\frac{\dot{n}''_i\lambda_{c,i}}{N_A}\frac{d\bar{u}_i(T)}{dT}\frac{\partial T}{\partial x} = -\frac{4}{3}\frac{\dot{n}''_i\lambda_{c,i}}{N_A}\bar{c}_{v,i}\frac{\partial T}{\partial x} = \dot{q}''_i$$

The proportionality between the rate of continuum heat transport and the local temperature gradient is normally reported in terms of the constitutive equation called the *Fourier-Biot Law*, $\dot{q}''_{x,i} = -K_i\partial T/\partial x$, that defines the thermal conductivity component, K_i

$$K_i = \frac{4\,\dot{n}''_i}{3\,N_A}\bar{c}_{v,i}\lambda_{c,i} = \frac{\rho_i\hat{V}_i c_{v,i}}{3}\lambda_{c,i} \qquad \text{(IIIE-20a)}$$

Note the similarity of the expressions for K_i and μ. In a fluid the carrier particle for both heat and momentum is the molecule. Thus the molecular conductivity of a fluid is

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[15]Note that there need be no net particle transport in the x direction in order for the y momentum to be transported in this direction.

[16]We will apply this relationship to molecule, phonon, and free electron carrier particles. The result is equally applicable to a photon continuum, or radiant transport in optically thick media. However, it is unusual for radiation to be an appreciable transport process in media that are optically thick.