But in the limit $\lim_{\varepsilon \Rightarrow 0}\left[\frac{\mathrm{erf}(\varepsilon)}{\varepsilon}\right]^{\frac{1}{2}} = 0$ $\left[\varepsilon = \left(\frac{\theta_{tr}}{T}\right) << 1\right]$

Thus only the first integral contributes. It is listed in Table IIIE-1. The result is

$$Z_{tr} = \left[\int_0^\infty e^{\zeta^2(-\theta_{tr}/T)}\, d\zeta\right]^3 = \frac{1}{8}\left(\frac{\pi T}{\theta_{tr}}\right)^{3/2}$$

The preceding problem, Exmp. 4, as well as Exer. A1 of Appendix IIIC-A developed Z_{tr} in the high-temperature limit.

$$Z_{tr} = \frac{1}{8}\left(\frac{\pi T}{\theta_{tr}}\right)^{3/2}$$

Substitution of the definition of $\theta_{tr}[\equiv h^2/(8m^* V^{2/3} k)]$ gives the partition function in terms of physical parameters. To obtain preferred molar units, we simply replace the total volume with the molar volume, $V = n\bar{v}$. Thus,

$$Z_{tr}(T, n\bar{v}) = n\bar{v}\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2} = n\bar{v}\left[\frac{2\pi M\bar{R}T}{(N_A h)^2}\right]^{3/2}$$

Now that Z_{tr} has been expressed analytically the property contributions of the translational mode are determinable. A few illustrations are presented in the following problems. Results are summarized in Table IIIC-1.

Exercise 3. Apply eqns. IIIC-3 and 7 to develop expressions for the translational internal energy and constant volume specific heat.

Example 5. Apply eqn. IIIC-5 to evaluate the translational contribution to pressure in a system of independent indistinguishable particles. Examine the contribution of the internal modes to the total pressure and determine the equation of state for the gaseous system.

The translational contribution to the pressure is, eqn. IIIC-5.

$$p_{tr} = \bar{R}T\left[\frac{\partial \ln[Z_{tr}(T, n\bar{v})]}{\partial \bar{v}}\right]_T = \bar{R}T\frac{\partial}{\partial \bar{v}}\ln\left[n\bar{v}\left(\frac{2\pi M\bar{R}T}{(N_A h)^2}\right)^{3/2}\right]_T = \bar{R}T\left(\frac{\partial}{\partial \bar{v}}\ln \bar{v}\right)_T$$

Recognizing that the internal partition function, $Z_{in}(T)$, is independent of volume,

$$p = p_{tr} + \cancel{p_{in}} = \frac{\bar{R}T}{\bar{v}}$$

We find that a system of independent translating particles in the Boltzmann limit is a perfect gas.

Exercise 4. Combine the results of Exer. 3 and Exmp. 5 to deduce the translational enthalpy.

Exercise 5. Use eqn. IIIC-11a and the results of Exer. 3 and Exmp. 5 to write an expression for the translational entropy in terms of temperature and pressure.

The solutions of the preceding problems demonstrate that once the partition function of a mode is known, the determination of its properties is relatively straightforward. In the