properties of a single macrostate.[22] Rather, the observed properties are the time average of many microstate and macrostate changes. If the time span is sufficient, the assumption of equal microstate probability will allow the calculation of the long time or *"ensemble" average properties* on the basis of a weighted average over the macrostates.[23] Each macrostate contributes to the ensemble average system property in proportion to its probability. We designate ensemble average properties with a (^), for either intensive or extensive properties.

$$\hat{\lambda} = \sum_{i=I}^{I^*} p_i \bar{\lambda}_i \quad \text{or} \quad \hat{\Lambda} = \sum_{i=1}^{I^*} p_i \bar{\Lambda}_i \qquad \text{(IIIA-11)}$$

Macrostate average properties, like macrostate distributions, are identical for all three statistics. However, ensemble average properties and hence observed properties vary with particle type for the same macrostate possibilities.

Example 10. Calculate the ensemble average pseudo-pressure $\hat{\varepsilon}^{\frac{1}{2}}$, for the 4 particle system of Boltzons, Table IIIA-2. (Five place accuracy will be required to establish differences.)

The macrostate probabilities, p_i^{MB} are listed in Table IIIA-3 and repeated in Table IIIA-4 along with macrostate average pseudo-pressures. Substitution into eqn. IIIA-11 gives

$$\hat{\varepsilon}^{\frac{1}{2}} = \sum_{i=1}^{5} p_i \overline{\varepsilon_i^{\frac{1}{2}}} = [0.537 \cdot 2.9784 + 0.179 \cdot 2.9205 + 0.134 \cdot 3.0000 + 0.090 \cdot 2.9568 + 0.060 \cdot 2.9151]\, B_o^{\frac{1}{2}} = 2.9649\, B_o^{\frac{1}{2}}$$

Exercise 11. Use the macrostate probabilities, p_i^{B-E} and p_i^{F-D}, and macrostate average pseudopressures listed in Table IIIA-4 to compute the ensemble average pseudopressure for a system of 4 Bosons and 4 Fermions. (The answers are listed in Table IIIA-4.)

TABLE IIIA-4: CHARACTERISTICS OF 4 PARTICLE SYSTEMS OF BOLTZONS, BOSONS, AND FERMIONS

Macrostate	$\bar{\varepsilon}_i^{\frac{1}{2}}/B_o^{\frac{1}{2}}$	p_i^{MB}	$\frac{\lvert\hat{\varepsilon}^{\frac{1}{2}} - \bar{\varepsilon}_i^{\frac{1}{2}}\rvert^{MB}}{\hat{\varepsilon}^{\frac{1}{2}}}$	p_i^{BE}	$\frac{\lvert\hat{\varepsilon}^{\frac{1}{2}} - \bar{\varepsilon}_i^{\frac{1}{2}}\rvert^{BE}}{\hat{\varepsilon}^{\frac{1}{2}}}$	p_i^{FD}	$\frac{\lvert\hat{\varepsilon}^{\frac{1}{2}} - \bar{\varepsilon}_i^{\frac{1}{2}}\rvert^{FD}}{\hat{\varepsilon}^{\frac{1}{2}}}$
I	2.9784	0.5373	0.4553%	0.3461	0.3741%	0.9	0.1951%
II	2.9205	0.1791	1.4982%	0.1921	1.5779%	0.1	1.7533%
III	3.0000	0.1343	1.1838%	0.2885	1.1020%	0	--
IV	2.9568	0.0895	0.2735%	0.1154	0.3542%	0	--
V	2.9151	0.0597	1.6810%	0.0577	1.7605%	0	--
		1.000	$\left(\frac{\hat{\varepsilon}^{\frac{1}{2}}}{B_o^{\frac{1}{2}}}\right)^{MB} = 2.9649$	1.000	$\left(\frac{\hat{\varepsilon}^{\frac{1}{2}}}{B_o^{\frac{1}{2}}}\right)^{BE} = 2.9673$	1.00	$\left(\frac{\hat{\varepsilon}^{\frac{1}{2}}}{B_o^{\frac{1}{2}}}\right)^{FD} = 2.9726$

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[22] As a specific illustration of the magnitude of the collision rate, in one cm^3 of air at STP there are on the order of 10^{28} collisions/s.

[23] A more formal treatment of statistical thermodynamics introduces the probability distribution in terms of the instantaneous condition of an ensemble of identical system replicas. Our isolated system model is known as a microcanonical ensemble. The equivalence between the instantaneous distribution of the ensemble and the long time trace of a single system is termed the ergodic hypothesis (see Ref. 7).