Example 8. Formulate the ratio K_{el}/σ_{el} by substitution of all known parameters into eqns. IIIE-20b and 24b. Verify that all unknown parameters cancel and obtain the result in terms of universal constants and temperature.

The ratio of interest is of the general form

$$\frac{K_{el}}{\sigma_{el}} = \frac{\rho\, V_F \dfrac{c_{v,el}}{3} \lambda_{c,F}}{\dfrac{n_e\, N_A\, e^2\, \lambda_{c,F}}{m_e^*\, M\, V_F}} = \frac{V_F^2\, M\, c_{v,el}\, m_e^*}{3\, n_e\, N_A\, e^2}$$

Then substituting from eqns. IIID-22 and IIIE-7

$$c_{v,el} = \frac{\pi^2}{2} M \bar{R}\left(\frac{T}{\theta_F}\right) \qquad V_F = \left(\frac{2k\,\theta_F}{m_e^*}\right)^{\frac{1}{2}} \qquad \bar{R} = N_A\, k$$

we have

$$\frac{K_{el}}{\sigma_{el}} = \frac{1}{3}\left(\frac{\pi k}{e}\right)^2 T$$

The preceding problem expressed the analogy between the thermal and electrical conductivity of free electrons in a metal in the analytic form known as the *Wiedemann-Franz Law*.

$$\frac{K_{el}}{\sigma_{el}\, T} = L = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 = 2.45 \cdot 10^{-8}\ \mathrm{W\Omega/K^2} \qquad \text{(IIIE-25)}$$

where L is known as the *Lorentz number*. The Wiedemann-Franz law is sufficiently accurate that the relatively easily measured electrical conductivity is often employed to predict the thermal conductivity of good conductors.[23]

Example 9. Platinum resistance thermometers are constructed of a carefully wound strain free coil of highly purified platinum wire. A particular thermometer uses a 3 m length of 50 μm diameter wire. Its calibration curve includes the following values: T = 10 K, R = 0.2723 Ω; T = 100 K, R = 42.36 Ω; and T = 300 K, R = 151.3 Ω. Use these data and the Wiedemann-Franz law to determine the thermal conductivity of platinum at each of these temperatures. In addition, determine the free electron mean free path at these temperatures.

The resistance of a wire is related to its electrical conductivity by $R = L/(A_c \sigma_{el})$. Thus σ_{el} can be determined from

$$\sigma_{el}(10\ \mathrm{K}) = \frac{4\,L}{\pi d^2 (10\ \mathrm{K})} = \frac{4 \cdot 3\ \mathrm{m}}{\pi \cdot (50 \cdot 10^{-6})^2\, \mathrm{m}^2 \cdot 0.2723\ \Omega} = 5.611 \cdot 10^9/(\Omega\mathrm{m})$$

Similarly, $\sigma_{el}(100\ \mathrm{K}) = 3.607 \cdot 10^7/(\Omega\mathrm{m})$ and $\sigma_{el}(300\ \mathrm{K}) = 1.010 \cdot 10^7/(\Omega\ \mathrm{m})$. The eqn. IIIE-25 gives the corresponding values for K_{el}

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[23]The Wiedemanj-Franz law only predicts the free electron contribution to thermal conductivity. Its utility is therefore restricted to good conductors in which $K_{el} >> K_{phn}$. The Wiedemann-Franz law is sufficient for engineering accuracy at both low and high temperatures. The relationship is not as accurate at intermediate temperatures in good conductors with relatively large values of $\lambda_{c,el\text{-}def}$.