Free electron properties are determined using eqn. IIID-21 and 22

$$\bar{u}^{\circ}_{el} = \frac{3}{5}\bar{R}\theta_F = \left(\frac{3}{5}\right)\cdot 8.314\ \text{J/(g mol K)}\cdot 8.16\cdot 10^4\text{K} = 4.07\cdot 10^5\text{J/(g mol)}$$

$$\bar{u}_{el}(100\ \text{K}) - \bar{u}^{\circ}_{el} = \bar{u}^{\circ}_{el}\cdot\left(\frac{5}{12}\right)\cdot\pi^2\left(\frac{T}{\theta_F}\right)^2 = 4.07\cdot 10^5\text{J/(g mol)}\cdot\left(\frac{5}{12}\right)\cdot\pi^2\left(\frac{100\ \text{K}}{8.16\cdot 10^4\text{K}}\right)^2 = 2.51\ \text{J/(g mol)}$$

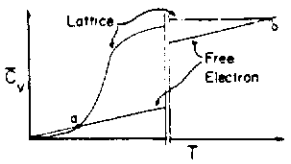
$$\bar{c}_{v,el} = \frac{\pi^2}{2}\bar{R}\left(\frac{T}{\theta_F}\right) = \frac{\pi^2}{2}\cdot 8.314\ \text{J/(g mol K)}\cdot\left(\frac{100\ \text{K}}{8.16\cdot 10^4\text{K}}\right) = 0.0503\ \text{J/(g mol K)}$$

The values at other temperatures are found similarly. The results are shown in tabular form.

T (K)	$\frac{(\bar{u}-\bar{u}^{\circ})_{phn}}{\bar{R}}$ (K)	$\frac{\bar{c}_{v,phn}}{\bar{R}}$	$\frac{(\bar{u}-\bar{u}^{\circ})_{el}}{\bar{R}}$ (K)	$\frac{\bar{c}_{v,el}}{\bar{R}}$	$\frac{\bar{u}-\bar{u}^{\circ}}{\bar{R}}$ (K)	$\frac{\bar{u}_{mtl}}{\bar{R}}$ (K)	$\frac{\bar{c}_{v,mtl}}{\bar{R}}$
100	80.4	1.95	0.301	$0.605\cdot 10^{-2}$	80.7	$4.94\cdot 10^4$	1.96
300	597	2.87	2.72	$1.82\cdot 10^{-2}$	600	$4.99\cdot 10^4$	2.89
1000	2660	3.00	30.1	$6.05\cdot 10^{-2}$	2690	$5.20\cdot 10^4$	3.06

Although Exmp. 10 treats a specific substance, it allows one to draw general conclusions. The zero point energy of the free electrons is a large component of the total energy of a metallic conductor. So much so that it masks the important effect of the variation of net energy with temperature. At ordinary temperatures, $\theta_D < T << \theta_F$, phonons make a dominant contribution to energy change or specific heats, although the free electron contribution is not negligible. At low temperatures ($T << \theta_D, \theta_F$) the phonon component of specific heat becomes proportional to T^3, but the free electron contribution remains linear. Thus as temperature is reduced below a certain level the electron contribution becomes more important and it will dominate at sufficiently low temperature levels.

Exercise 17. The linear temperature dependence of $\bar{c}_{v,el}$ assures that it will dominate $\bar{c}_{v,phn}$ at both sufficiently low and, in theory, at sufficiently high temperatures. Determine both the low and high temperatures, T_a and T_b, where $\bar{c}_{v,phn} = \bar{c}_{v,el}$ for solid silver.



Ans. 16. The necessary valences and molecular weights can be extracted from Table IA-1 and the densities from Table ID-2.

Al, $n_e = 3$, M = 26.98 g/(g mol), $\rho = 2.70$ g/cm^3; Cu, $n_e = 1$, M = 63.55 g/(g mol), $\rho = 8.94$ g/cm^3; Pb, $n_e = 2$, M = 207.2 g/(g mol), $\rho = 11.34$ g/cm^3. Thus using eqn. IIID-20

$$\theta_{F,Al} = \frac{(6.625\cdot 10^{-34}\text{Js/elect})^2}{8\cdot 9.109\cdot 10^{-31}\text{kg}\cdot 1.380\cdot 10^{-23}\text{J/elect K}}\cdot\left[\frac{3\cdot 3\ \text{elect/part}\cdot 2.70\cdot 10^3\text{kg/m}^3\cdot 6.023\cdot 10^{26}\text{part/kg mol}}{\pi\cdot 26.98\ \text{kg/(kg mol)}}\right]^{2/3} = 1.35\cdot 10^5\text{K}$$

Similarly, $\theta_{F,Cu} = 8.16\cdot 10^4$K, $\theta_{F,Pb} = 6.91\cdot 10^4$K.