III. $n_{1,III} = 2;\quad n_{2,III} = n_{3,III} = \ldots = n_{9,III} = 0;\quad n_{10,III} = 1$

Check: $2 \cdot 3\ B_o + 0 \cdot 6\ B_o + \ldots + 1 \cdot 21\ B_o = 27\ B_o$

This macrostate has bracketed the potentially occupied levels. No higher levels can be occupied in this system. Also, that constitutes all the macrostates with two particles in the same state.

An additional macrostate is found by reexamining macrostate I. If one particle's energy is reduced by 3 B_o and one increased 3 B_o, we have macrostate IV.

IV. $n_{2,IV} = n_{3,IV} = n_{5,IV} = 1;\quad n_{1,IV} = n_{4,IV} = n_{6,IV} = \ldots = 0$

Check: $0 + 1 \cdot 6\ B_o + 1 \cdot 9\ B_o + 0 + 1 \cdot 12\ B_o + 0 = 27\ B_o$

Similarly consider macrostate III if one of the particles in the lowest energy is increased 3 B_o and the other is reduced to level 8, we have macrostate V.

V. $n_{1,V} = n_{2,V} = n_{8,V} = 1;\quad n_{3,V} = n_{4,V} = \ldots = n_{7,V} = n_{9,V} = 0$

Because we have used a patterned method of looking, it is clear that no others exist. These are the five macrostates of our example.

The trial-and-error determination of macrostates for 3 particles is relatively easy because our selection of system energy limited the potentially occupied levels to only the lowest ten of the infinite array. In the course of the unit we will consider the characteristics of systems with successively larger numbers of particles and eventually extrapolate to draw conclusions about real systems. In the process, to maintain as much commonality as possible throughout the systems of our study, we choose to hold the average energy per particle, $\bar{\varepsilon}$, fixed at a constant value, $\bar{\varepsilon} = U/N = 9\ B_o$. Therefore as we proceed to larger systems the total energy will increase in proportion. What about the number of levels accessible to the system?

In the 3 particle example the 10th was the highest level occupied. For a 30 particle example the total energy is 270 B_o. This means that the highest level occupied would be $\varepsilon_{129} = 180\ B_o$ ($n_{1,I} = 28$, $n_{2,I} = 1$, $n_{129,I} = 1$). Thus the number of levels accessible to particles increases very rapidly with increased total energy (or system size). This in turn, vastly increases the number of macrostates. A 4 particle system would have 14 possible energy levels and 14 macrostates. Since our principal purpose is to investigate increased numbers of particles we will facilitate the process by arbitrarily limiting our systems to the lowest five levels, Fig. IIIA-1. In the figure, states and levels of increased energy are placed higher on the page in analogy with greater energy. The existence of more than one energy

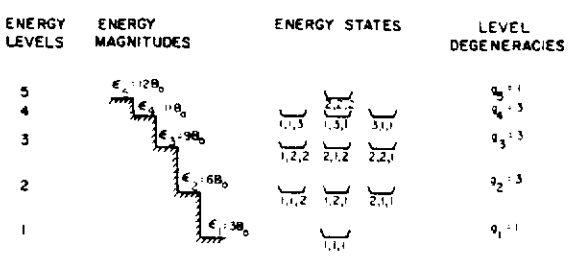

Fig. IIIA-1: A Schematic Representation of the Energy States of a 5-Energy Level System