Exercise 15. Determine the volume that would be required to achieve a free electron translational characteristic temperature of 1 K.

Exer. 15 demonstrates that although electrons are far lighter than even the lightest molecules, θ_{tr} for free electrons remains low enough that free electron energy can also be considered continuous. Therefore, the free electron distribution can be written in terms of the degeneracy developed for the translation of independent molecules in Appendix

Ans. 12. Phonons are similar to photons in that: $\alpha = 0$, $\varepsilon_j = h\nu_j$, and the $\sum_j \Rightarrow \int d\nu$. Thus the general form of eqn. IIIB-17 reduces to

$$S_{phn} = \eta\bar{s}_{phn} = \frac{\eta\bar{u}_{phn}}{T} - k\int_0^\infty g_{phn}(\nu)\ln\left(1 - e^{-h\nu/kT}\right)d\nu$$

Then substituting from eqns. IIID-12a and 13b

$$S_{phn} = \eta\bar{s}_{phn} = \eta 3\bar{R}D(x_D) - \frac{12\pi Vk}{c_s^3}\int_0^{\nu_D} \nu^2 \ln\left(1 - e^{-h\nu/kT}\right)d\nu$$

When the integral term is expressed in dimensionless form and transformed using the same definitions for integration by parts as defined in Exmp. 3, we obtain

$$-\frac{12\pi Vk}{c_s^3}\left(\frac{kT}{h}\right)^3\int_0^{\theta_D/T} x^2\ln(1 - e^{-x})dx = -12\pi Vk\left(\frac{kT}{hc_s}\right)^3\left\{\left[\frac{x^3}{3}\ln(1 - e^{-x})\right]_0^{\theta_D/T} - \frac{1}{3}\int_0^{\theta_D/T}\frac{x^3}{e^x 01}dx\right\}$$

Then if we introduce the definition of θ_D in the form

$$\theta_D^3 = \left(\frac{hc_s}{k}\right)^3\frac{3N}{4\pi V} = \left(\frac{hc_s}{k}\right)^3\frac{3\eta N_A}{4\pi V}$$

we obtain

$$S_{phn} = \eta\bar{s}_{phn} = \eta 3\bar{R}D(x_D) - 9\eta N_A k\left(\frac{T}{\theta_D}\right)^3\left[\frac{x_D^3}{3}\ln\left(1 - e^{-x_D}\right) - \frac{1}{3}\int_0^{x_D}\frac{x^3dx}{e^x - 1}\right]$$

which gives eqn. IIID-16 using $\bar{R} = N_A k$ and the definition of $D(x_D)$

Ans. 13. $\Theta_D = 396$ K. Thus at 100 K, $x_D = 396$ K/100 K = 3.96. And from Table IIID-2, D(3.96) = 0.186. Substituting we obtain

$$\bar{u}_{phn} = 3\bar{R}[TD(x_D)] = 3 \cdot 8.314 \text{ J/(g mol K)} \cdot (100 \text{ K} \cdot 0.186) = 464 \text{ J/(g mol)}$$

$$\bar{c}_{v,phn}(x_D) = 3\bar{R}\left[4\,D(x_D) - \frac{3x_D}{(e^{x_D} - 1)}\right] = 3 \cdot 8.314 \text{ J/(g mol K)} \cdot \left[4 \cdot 0.186 - \frac{3 \cdot 3.96}{(e^{3.96} - 1)}\right]$$

$$= 12.8 \text{ J/(g mol K)}$$

$$\bar{s}_{phn} = \bar{R}\left[4\,D(x_D) - 3\ln\left(1 - e^{-x_D}\right)\right] = 8.314 \text{ J/(g mol K)} \cdot \left[4 \cdot 0.186 - 3\ln\left(1 - e^{-3.96}\right)\right]$$

$$= 6.67 \text{ J/(g mol K)}$$

In a similar manner find

T(K)	x_D	$D(x_D)$	$\frac{\bar{u}_{phn}(x_D)}{R\Theta_D}$	$\frac{\bar{c}_{v,phn}(x_D)}{\bar{R}}$	$\frac{\bar{s}_{phn}}{\bar{R}}$
100	3.96	0.186	0.140	1.53	0.802
300	1.32	0.600	1.36	2.87	3.33
1000	0.396	0.860	6.52	2.98	6.79