its limits explicitly in terms of θ_p. In this form the general series is

$$Z_p = \sum_{\ell=1}^{L} g_{p,\ell}\, e^{-[f_p(\ell)\theta_p]/T} \qquad \text{(IIIC-14a)}$$

And the low- and high-temperature limits are

$$Z_p\Big|_{T<<\theta_p} = g_{p,1}\, e^{-[f_p(1)\theta_p]/T} + g_{p,2}\, e^{-[f_p(2)\theta_p]/T} \qquad \text{(IIIC-14b)}$$

$$Z_p\Big|_{T>>\theta_p} = \int_{\ell=1}^{L} g_p(\ell)\, e^{-[f(\ell)\theta_p]/T}\, d\ell \qquad \text{(IIIC-14c)}$$

These relationships are applied to the external and internal modes of gaseous particles in the following section.

III. THE CHARACTERISTICS OF ENERGY MODES

The characteristics of independent indistinguishable particles can be separated into a single external and several internal modes. In this section we develop expressions for their individual partition functions and property contributions. We begin with the external mode.

A. The External (Translational) Mode

The external energy of independent, indistinguishable particles results from translation. The quantum mechanics of independent particle translation were treated in Unit IIIA. In the form of eqns. IIIC-12 and 13 (see Exer. 1) the translational energy states are

$$\varepsilon_{tr}(\ell,m,n) = k\theta_{tr}(m^*,V)\cdot f(\ell,m,n)$$

where $\theta_{tr}(m^*,V) = \dfrac{h^2}{8m^*V^{2/3}k}$ $\quad f(\ell,m,n) = (\ell^2+m^2+n^2)$ $\quad \ell,m,n = 1,2,3,\ldots,$

Notice that $\theta_{tr}(m^*,V)$ is dependent on the mass of the translating particles and the extensive volume of their container (not the molar volume). To establish a quantitative feel for the range of validity of the high and low temperature limits of this mode consider the following example.

Example 3. Determine the magnitude of θ_{tr} for nitrogen gas in a 30 m^3 room. Particle mass is determined from definition 7 of Unit IA.

$$m^* = \frac{M}{N_A} = \frac{28.1\ \text{kg/(kg mol)}}{6.023\cdot 10^{26}\ \text{part/(kg mol)}} = 4.649\cdot 10^{-26}\ \text{kg/part}$$

Thus the characteristic temperature is (see Table IA-2 for the required physical constants)

$$\theta_{tr} = \frac{h^2}{8m^*V^{2/3}k} = \frac{(6.625\cdot 10^{-34}\ \text{Js/part})^2\cdot 1\ \text{kg m}^2/(\text{Js}^2)}{8\cdot 4.649\cdot 10^{-26}\ \text{kg/part}\ (30\ \text{m}^3)^{2/3}\cdot 1.38\cdot 10^{-23}\ \text{J/(part K)}} = 8.857\cdot 10^{-21}\ \text{K}$$

Exmp. 3 demonstrates that θ_{tr} is extremely small for nitrogen molecules in a room size