Example 1. Consider a jet engine in steady level flight at velocity $\hat{V}_o$ relative to the atmosphere. Perform energy analyses of the air that flows through the engine using two different reference frames, one fixed to the engine and the other fixed to the space through which the aircraft flies. Subtract these two expressions to show that the work done by the engine is equal to the product of its forward velocity, $\hat{V}_o$, times its developed *thrust* per mass of flow, $(|\hat{V}_e - \hat{V}_o|)$.

Using the reference frame fixed to the engine, and assuming $r_{f/a} = 0$, the steady state energy equation is

$$0 = \left(h_o + \frac{\hat{V}_o^2}{2}\right) - \left(h_e + \frac{\hat{V}_e^2}{2}\right) + |q_H|$$

In this reference frame the system work is zero, since there is no forward velocity relative to the frame.

In a reference frame fixed to the space through which the system moves, the velocity of the gas entering the engine is zero. The exit velocity in this reference frame is the absolute velocity, $|\hat{V}_{e.abs}| = |\hat{V}_e| - |\hat{V}_o|$. The energy equation is

$$0 = h_o - \left(h_e + \frac{|\hat{V}_e - \hat{V}_o|^2}{2}\right) + |q_H| - |w_s| = 0$$

The properties of the media are unaffected by the reference frame. These include the enthalpies and the energy released by the combustion process, $|q_H|$. Thus the two energy equations can be subtracted to determine system work in terms of velocities

$$w_{s,j} = -\frac{|\hat{V}_e - \hat{V}_o|^2}{2} + \frac{\hat{V}_e}{2} - \frac{\hat{V}_o}{2} = \hat{V}_o\hat{V}_e - \hat{V}_o^2 = \hat{V}_o(|\hat{V}_e| - |\hat{V}_o|) \qquad \text{Q.E.D.}$$

The mass of a rocket varies with its expenditure of propellant. Thus the vehicle as a system does not satisfy steady-state conditions. The rocket engine, as a part of the vehicle, intakes propellants and combusts them to generate hot gas products that are then expelled from its nozzle as a high velocity jet. Therefore, it is possible to analyze the *rocket engine* as a *steady state* system.

Exercise 1. Modify the steady state energy equations for an air-breathing engine (see Exmp. 1) to the conditions of a steady state rocket engine. Subtract them to derive the following expression for the work done by the rocket.

$$|w_{s,r}| = \hat{V}_o\hat{V}_e$$

Exmp. 1 and Exer. 1 developed the system work per mass of flow through jet and rocket engines respectively from an energy perspective. System work can also be predicted from the momentum principle. The force developed by a propulsion engine equals the rate of change of momentum of the gases that move through it. For jet and rocket engines, the propulsion forces are, respectively,[3]

$$F_j = F_{thr.,j} - (p_e - p_o)A_e = \dot{m}(\hat{V}_e - \hat{V}_o) \qquad \text{(VD-2a)}$$

[3]The assumption that $r_{f/a} = 0$ is incorporated in eqn. VD-2a. In general it would be $F_j = \dot{m}_a[(1 + r_{f/a})V_e - V_o]$.