$$n_j^{MB} = \frac{g_j}{e^{[\alpha+\varepsilon_j(V)]/kT}} \qquad n_j^{FD} = \frac{g_j}{e^{[\alpha+\varepsilon_j(V)]/kT}+1} \qquad n_j^{BE} = \frac{g_j}{e^{[\alpha+\varepsilon_j(V)]/kT}-1} \qquad \text{(IIIB-8,14,16)}$$

Inspection reveals that these forms differ by a single term, namely, the +1, -1, or 0 appearing in the denominator. Therefore the two indistinguishable particle distribution functions become identical to one another and to the distinguishable particle, Maxwell-Boltzmann distribution function in the limit $e^{[\alpha+\varepsilon_j(V)]/kT} \gg 1$. This limit, called the *indistinguishable* or the Boltzmann *limit* (*il*), is described by the condition $n_j \ll g_j$.

Recall that the distinction between Fermions and Bosons results from the Pauli exclusion principle. (No more than one Fermion can occupy a state, $n_j^{FD} < g_j$; but any number of Bosons can.) But the indistinguishable limit is $n_j \ll g_j$. Therefore, in this limit very few states would contain more than one Boson and the effect of the Pauli exclusion principle on the distribution is inconsequential. This provides a physical explanation for the fact that $n_j^{FD} \simeq n_j^{BE} = n_j^{il}$ when $n_j \ll g_j$. The following example provides an analytic explanation.

Example 1. Use the condition $n_j \ll g_j$ to establish a relationship between the number of microstates in a macrostate, W_i, for indistinguishable Fermions and Bosons to that of distinguishable Boltzons valid in the indistinguishable limit.

Eqns. IIIA-7a, b and c give the relationships between $n_{j,i}$, g_j and W_i. For Fermions it is $W_i^{FD} = \prod_j g_j!/[(g_j - n_{j,i})!n_{j,i}!]$. To establish its value in the limit, examine the ratio

$$\frac{g_j!}{(g_j-n_{j,i})!} = \frac{g_j(g_j-1)(g_j-2)\ \ldots\ (g_j-n_{j,i}+2)(g_j-n_{j,i}+1)(g_j-n_{j,i})!}{(g_j-n_{j,i})!}$$

But in the limit $n_{j,i} \ll g_j$, $\qquad g_j \simeq (g_j-1) \simeq \cdot \simeq \cdot \simeq (g_j-n_{j,i}+1)$

So

$$\frac{g_j!}{(g_j-n_{j,i})!} \simeq g_j^{n_{j,i}} \qquad \text{and} \qquad W_i^{FD} \Rightarrow \prod_j \frac{g_j^{n_{j,i}}}{n_{j,i}!}$$

The expression for Bosons is $W_i^{BE} = \prod_j (g_j + n_{j,i} - 1)!/[(g_j - 1)!n_{j,i}!]$. Using the same approach, examine

$$\frac{(g_j+n_{j,i}-1)!}{(g_j-1)!} = \frac{(g_j+n_{j,i}-1)(g_j+n_{j,i}-2)\ \ldots\ g_j(g_j-1)!}{(g_j-1)!}$$

But in the limit $n_{j,i} \ll g_j$ $\qquad (g_j+n_{j,i}-1) \simeq (g_j+n_{j,i}-2) \simeq .. \simeq g_j$

So

$$\frac{(g_j+n_{j,i}-1)!}{(g_j-1)!} \simeq g_j^{n_{j,i}} \qquad \text{and} \qquad W_i^{BE} \Rightarrow \prod_j \frac{g_j^{n_{j,i}}}{n_{j,i}!}$$

Further for Boltzons, $W_i^{MB} = N!\ \prod_j \left(g_j^{n_{j,i}}\right)/n_{j,i}!$. Thus when $n_{j,i} \ll g_j$, $W_i^{FD} \Rightarrow W_i^{BE} \Rightarrow W_i^{MB}/N!$