Upon substitution for jet and rocket engines respectively, *propulsive efficiency* becomes

$$\eta_{p,j} = \frac{2|\hat{V}_o/\hat{V}_e|}{1 + |\hat{V}_o/\hat{V}_e|} \qquad \eta_{p,r} = \frac{2|\hat{V}_o/\hat{V}_e|}{1 + |\hat{V}_o/\hat{V}_e|^2} \qquad \text{(VD-4a,b)}$$

The forms of eqns. VD-4 reveal some important operating characteristics of propulsion systems. Optimum propulsive efficiency occurs when the exit gas speed equals the airframe speed, that is, $\eta_p = 1$ when $|\hat{V}_e| = |\hat{V}_o|$. This optimum condition corresponds to zero exit kinetic energy relative to the ground. It has only theoretical significance to an air-breathing engine, since the engine would then develop no thrust (eqn. VD-2a). The thrust developed by a rocket engine is proportional to its exit velocity. A rocket develops thrust even when its exit velocity is less than the ground speed, $|\hat{V}_e| < |\hat{V}_o|$. This is an essential characteristic for interstellar travel when the speed of the vehicle must be very high. The *overall efficiency* of a propulsion system accounts for both its thermal efficiency in developing kinetic energy change and its mechanical or propulsion efficiency in converting that energy to propulsion power

$$\eta = \eta_{th}\eta_p \qquad \text{(VD-5)}$$

We explore the operating characteristics of a few jet and rocket engine cycles in the following sections.

II. JET ENGINES

Air-breathing jet engines are the principal means of aircraft propulsion at the high subsonic speeds used by most commercial aircraft and in the transonic and supersonic speeds employed by many military and some commercial aircraft. Jet engines employ a number of variations of a Brayton cycle (see Unit VA). We begin with a description of the processes that occur in an *idealized turbojet engine,* Fig. V-1.

States 0-1; isentropic deceleration of the entering air, initial speed V_o, as it approaches the engine and enters the DIFFUSER compressing the air from p_L to p_1. States 1-2; continued isentropic compression in the COMPRESSOR to p_H. States 2-3; constant pressure heat addition (fuel addition followed by combustion) in the COMBUSTOR to attain T_H. States 3-4; isentropic expansion in

Ans. 1. In the reference frame fixed to the engine the entering velocity is negligible

$$0 = h_o - \left(h_e + \frac{\hat{V}_e^2}{2}\right) + |q_H|$$

and in the reference frame fixed to the space, the inlet gas has a velocity that approaches $\hat{V}_o$. Hence

$$0 = \left(h_o + \frac{\hat{V}_o^2}{2}\right) - \left(h_e + \frac{|\hat{V}_e - \hat{V}_o|^2}{2}\right) + |q_H| - |w_{s,r}|$$

Subtracting, we obtain

$$|w_{s,r}| = -\frac{|\hat{V}_e - \hat{V}_o|^2}{2} + \frac{\hat{V}_e^2}{2} + \frac{\hat{V}_o^2}{2} = \hat{V}_o\hat{V}_e \qquad \text{Q.E.D.}$$