Indistinguishable particle macrostate probabilities. We found that most of the microstates of Maxwell-Boltzmann systems are simply interchanges of position of the distinguishable particles (A, B, ...) among the energy states. For a system of indistinguishable particles (a liquid or gas) particle position swapping does not result in new distinct distributions. Therefore, we expect fewer microstates for indistinguishable systems. In our previous consideration of distinguishable particles (Boltzons), we placed no limit on the number of particles that could occupy a state. There are two classifications of indistinguishable particles. One shares this characteristic with Boltzons. The other is restricted to at most one particle per state.

> The Pauli Exclusion Principle and its Significance. Indistinguishable particles are classed as symmetric or antisymmetric depending upon whether they contain an even or an odd number of subatomic components (that is, the total number of electrons, protons, and neutrons). The necessity for this distinction arises from the experimental fact that particles with an even number of subatomic components (symmetric particles) behave differently from those with an odd number of subatomic components (antisymmetric particles).[15] These differences can be accounted for by the postulate that each energy state of systems of antisymmetric particles is reserved for at most one particle at any instant, whereas any number of symmetric particles can occupy the same energy state simultaneously. This postulate is called the Pauli Exclusion Principle. It is an effect of the symmetry of the individual particle wave functions and the manner in which they combine to give the group wave function. If the individual wave functions are odd and two particles are placed in one state, the group wave function and therefore the probability of such an arrangement, vanishes. If the individual particle wave functions are even and two particles are placed in one state, the group wave function does not vanish. To make this point more rigorously, it is necessary to study the effect of matrix multiplication of individual wave functions to obtain group results. The interested student may wish to read Ref. 5, Sect. 5.10.

For application we require only knowledge of the additional restriction. Antisymmetric indistinguishable particle systems can have no more than one particle per state. Symmetric indistinguishable particle systems like distinguishable particles (Boltzons) can have any number of particles per state. Two statistical methods have been developed to predict the behaviors of the two classes of indistinguishable particles. These are *BOSE-EINSTEIN STATISTICS* for symmetric particles and *FERMI-DIRAC STATISTICS* for antisymmetric particles. Applications to systems of indistinguishable particles require that we be able to differentiate systems of symmetric particles, *Bosons*, from systems of antisymmetric particles, *Fermions*. This differentiation is accomplished by simply counting the number of subatomic components composing the particle, i.e., its total number of electrons, protons, and neutrons. The procedure is the same for all particles whether they be molecules, atoms, or electromagnetic and acoustical waves.

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[15]We will subsequently find (Unit IIIC) that the differences vanish in each system at its own characteristic high temperature.