state at a level is represented by several cups at the level. Each cup represents a state, they are subscripted with their quantum numbers to indicate that they are distinct.

Exercise 5. Determine the macrostates of a system of 3 particles, energy 27 B_o, when the levels are limited to 3 B_o, 6 B_o, 9 B_o, 11 B_o, and 12 B_o, as shown in Fig. IIIA-1.

Exercise 6. Given a system with 4 particles and 36 B_o energy to which only the five lowest energy levels of Table IIIA-1 are accessible, write the constraint equations and determine the macrostate distributions. (Answers in Table IIIA-2.)

TABLE IIIA-2: THE MACROSTATES OF A 4 PARTICLE, 36 B_o ENERGY SYSTEM WITH 5 ENERGY LEVELS

Macrostate	j	= 1	2	3	4	5	$\sum_{j=1}^{5} n_{j,i}$	$\sum_{j=1}^{5} n_{j,i}\varepsilon_j$
I	$n_{j,I}$	= 0	1	2	0	1	4	$36B_o$
II	$n_{j,II}$	= 1	0	0	3	0	4	$36B_o$
III	$n_{j,III}$	= 0	0	4	0	0	4	$36B_o$
IV	$n_{j,IV}$	= 0	2	0	0	2	4	$36B_o$
V	$n_{j,V}$	= 1	0	1	0	2	4	$36B_o$
	ε_j	= $3B_o$	$6B_o$	$9B_o$	$11B_o$	$12B_o$		

B. Microstates

In the three and four particle examples we found several solutions to the constraint equations, IIIA-6, i.e., several macrostates. Since each macrostate constitutes a distinct arrangement of the particles among the levels, we could expect the different macrostates to display different characteristics or properties. As collisions occur and macrostates change very rapidly, time average properties will be associated with macrostate properties in proportion to the probability of existence of each macrostate. Therefore we must be able to predict the probability of allowed macrostates.

A macrostate description specifies only the distribution of particles among levels. But most levels contain several states (those with $g_j \geq 2$). Therefore, most macrostates encompass several arrangements of particles among the states in the degenerate levels. Since these arrangements are elements of a macrostate set, they are called *microstates* of that macrostate. A *microstate* is a *unique arrangement* of particles *among the energy states* which satisfies the system constraints (eqns. IIIA-6). While macrostate determination is independent of particle characteristics (for given N, U, and ε_j), microstate counting procedures vary with the type of particle. We will first study distinguishable particles.

Distinguishable vs. Indistinguishable Particles. One cannot distinguish between like microscopic particles. One nitrogen atom is the same as any other nitrogen atom.[13] Nor can we tell one copper atom from another. Microscopic particles appear identical. However, within a crystal of copper we can distinguish the atoms by virtue of their fixed positions. This is the distinguishability of particles. The like particles constituting the lattice points of solids are dis-

[13]We exclude other isotopes.