Exercise 8. The average free electron energy was determined in Exmp. 9 of Unit IIID to be $\hat{\varepsilon}_{el} = (3/5)\varepsilon_F^\circ$. Verify that this result is consistent with eqn. IIIE-7b.

Exercise 9. Use the distribution of free electrons and the definition of a mean, eqns. IIIE-6 and 4, to determine the mean free electron speed, eqn. IIIE-7b.

Exercise 10. The Fermi temperature of the free electrons of solid copper is $8.44 \cdot 10^4$K, see Exer. 17 of Unit IIID. Determine the average, rms, and Fermi speed of these electrons.

Now that we have considered the speed distribution of molecular and free electron gases, the simplicity of the distribution applicable to photons and phonons is readily appreciated. Each electromagnetic and acoustic wave travels at the speed of light and sound respectively.

$$\rho_{pht}(V)\Big|_{V=c} = 1 \qquad \rho_{pht}(V)\Big|_{V\neq c} = 0 \qquad \text{(IIIE-9a)}$$

$$\rho_{phn}(V)\Big|_{V=c_s} = 1 \qquad \rho_{phn}(V)\Big|_{V\neq c_s} = 0 \qquad \text{(IIIE-9b)}$$

Thus their most probable, mean, and root mean square speeds all have the same value. For photons and phonons, respectively, the values are

$$V_{mp} = \hat{V} = V_{rms} = c \qquad V_{mp} = \hat{V} = V_{rms} = c_s \qquad \text{(III-10a,b)}$$

In the following section we use the speed distributions developed in this section to determine the maximum possible transport flux.

B. Interparticle Flux

The preceding section considered the motion of the four principal transport carriers. We found that their speed distributions varied substantially. However, all of these gases share one characteristic— the particles move in random directions. One result of this isotropicity of particle velocities is that there is no *net* transport when the carrier gas is in equilibrium. Another is that all of the gases have the same proportion of their collective speeds directed positively in any single direction.

Example 4. Determine the average velocity component of equilibrium gas particles contributing positively to transport in any single direction. Show that this average can be expressed as a fraction of the mean speed within the isotropically translating particles.

If we represent velocity in spherical coordinates $0 \le V < \infty$, $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, the projection of the general velocity vector in the direction $\theta = 0$ is $V\cos\theta$. The average of these components in the positive sense is determined by restricting the integral limits on θ to $0 \le \theta \le \pi/2$. Thus, the average positive velocity component is written symbolically as

$$\widehat{V\cos\theta}\Big|_{pos} \equiv \frac{\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\infty} V\cos\theta\,\rho(V,\theta,\phi)V^2\sin\theta\,dV\,d\theta\,d\phi}{\int_0^{2\pi}\int_0^{\pi/2}\int_0^{\infty} \rho(V,\theta,\phi)V^2\sin\theta\,dV\,d\theta\,d\phi}$$