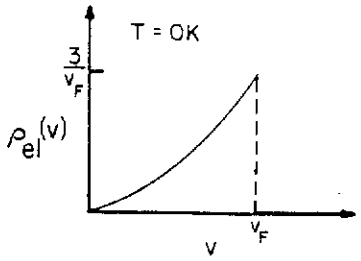
$$\rho_{el}(\varepsilon)d\varepsilon = \rho_{el}(V)dV\Big|_{V\le V_F} = \frac{3\left(\frac{m_e^* V^2}{2}\right)^{\frac{1}{2}}}{2\left(\frac{m_e^* V_F^2}{2}\right)^{3/2}} m_e^* VdV = 3\frac{V^2}{V_F^3}dV$$



and $\rho_{el}(V)\Big|_{V>V_F} = 0.$

In Exmp. 3 we derived the speed distribution for free electrons at zero temperature. This result is a good approximation for the electron speeds at all practical temperatures in a solid, $T \ll \theta_F$

$$\rho_{el}(V)\Big|_{V\le V_F} = 3\frac{V^2}{V_F^3} \qquad \rho_{el}(V)\Big|_{V>V_F} = 0 \qquad \text{where} \quad V_F \equiv \left(\frac{2k\theta_F}{m_e^*}\right)^{\frac{1}{2}} \qquad \text{(IIIE-7)}$$

Since the approximate distribution is independent of temperature, the *most probable, average, and rms speeds of free electrons* are constant for each substance.[8]

$$V_{mp} = V_F \qquad \hat{V} = \frac{3}{4}V_F \qquad V_{rms} = \sqrt{\frac{3}{5}}V_F \qquad \text{(IIIE-8a,b,c)}$$

[8]The highest probability is that at the Fermi level (see the figure of Exmp. 3). But the probability curve does not have zero slope at that point.

Ans. 5. The variation in the various average speeds are due to different molecular weights: $M_{N_2} = 28.01$, $M_{H_2O} = 18.02$, $M_{He} = 4.00$, $M_{C_8H_{18}} = 114.2$. Thus for nitrogen

$$V_{mp} = \sqrt{2RT} = \left[2\cdot\left(\frac{8314}{28.01}\right) J/(kg\,K)\cdot 300\ K\cdot 1\ kg\ m^2/(J\ s^2)\right]^{\frac{1}{2}} = 422.0\ m/s$$

and $V_{rms} = \sqrt{\frac{3}{2}}V_{mp} = 516.8$ m/s

Similarly V_{mp} and V_{rms} for the other molecules are respectively: H_2O - 526.3 m/s and 644.4 m/s, He - 1117 m/s and 1368 m/s, C_8H_{18} - 209.0 m/s and 256.0 m/s.

Ans. 6. For the conditions specified

$$x^* = \frac{V^*}{\sqrt{2RT}} = \frac{1000\ m/s}{\left[2\cdot\left(\frac{8314}{32.00}\right) J/(kg\ K)\cdot 600\ K\cdot 1\ kg\ m^2/(J\ s^2)\right]^{\frac{1}{2}}} = 1.79$$

Interpolating from Table IIIE-1, we find $\int_0^{x^*} x^2 e^{-x^2} dx = 0.401$. And

$$P(0 \to x^*) = \frac{4}{\sqrt{\pi}}\int_0^{x^*} x^2 e^{-x} dx = \frac{4}{\sqrt{\pi}}\cdot 0.401 = 0.906$$

The probability is that 9.4% of the molecules have speeds above 1000 m/s.

Ans. 7. In dimensionless form $x_{mp} = V_{mp}/\sqrt{2RT} = 1$ and $x_{rms} = V_{rms}/\sqrt{2RT} = \sqrt{3/2} = 1.225$. Thus, $P(0 - x_{mp}) = 0.1895$ and $P(0 - x_{rms}) = 0.2687$. So (26.9 - 19.0) or 7.9% of the particles are between these speeds.