Example 3. Electrical conduction in metals is dependent on the motion and energy exchange processes of its free electrons. Free electrons can be modeled as point particles in a field free box. The box in this case is the conducting element. The energy levels in a circular wire can be approximated as

$$\varepsilon \sim \frac{h^2}{8m_e^*}\left(\frac{\ell^2 + m^2}{D^2} + \frac{n^2}{L^2}\right)$$

where D is the diameter and L the length of the wire. Scattering processes that change the radial component of the velocity will necessarily involve a change in the quantum numbers ℓ and/or m. Calculate the minimum change in the electron energy for such a process and the corresponding discrete change in the electron's translational velocity for a 10-micron diameter wire.

The minimum discrete energy change would correspond to one integer in the magnitude of the ℓ and m quantum numbers squared, $(\ell^2 + m^2)_2 - (\ell^2 + m^2)_1 = 1$

$$\Delta\varepsilon_{min} = \frac{h^2}{8m_e^* D^2} = \frac{(6.625)^2 \cdot 10^{-54}\ \text{erg}^2\text{s}^2}{8 \cdot (9.109) \cdot 10^{-28}\ \text{g} \cdot 10^{-6}\ \text{cm}^2} = 6.023 \cdot 10^{-21}\ \text{erg}$$

The relationship to the velocity is in terms of the kinetic energy of the electron $\varepsilon = m_e^*(\hat{V}^2/2)$; so

$$\Delta\hat{V}_{min} = \left(\frac{2\Delta\varepsilon_{min}}{m_e^*}\right)^{\frac{1}{2}} = \frac{h}{2Dm_e^*} = \frac{6.625 \cdot 10^{-27}\ \text{erg-s}}{2 \cdot 10^{-3}\ \text{cm} \cdot 9.109 \cdot 10^{-28}\ \text{g}} = 3{,}637\ \text{cm/s}$$

Thus, though the discrete energy change is numerically small we nevertheless obtain large minimum radial velocity change. It is clear from this example that quantum mechanics is significant in determining electrical conductivity.

Exercise 3. While discrete energy changes are significant for some microscopic particles, they are insignificant for all macroscopic objects. Consider a billiard ball moving on a *frictionless* table. A frictionless billiard table can be modeled as a two-dimensional field-free box. The energies of the balls in such a system can be inferred from eqn. IIIA-4 to be

$$\varepsilon_{2\text{-dim.}} = \frac{h^2}{8m^*}\left(\frac{\ell^2}{L_x^2} + \frac{m^2}{L_y^2}\right)$$

Ans. 2. The function, (x,y,z), satisfies the boundary conditions at the limits of x for all y and z. At x = 0

$$\psi_{\ell,m,n}(0,y,z) = A \sin\left(\frac{\ell\pi}{L_x} \cdot 0\right) \sin\left(\frac{m\pi y}{L_y}\right) \sin\left(\frac{n\pi z}{L_z}\right)$$

which is zero, since sin (0) = 0. At $x = L_x$ we have

$$\psi_{\ell,m,n}(L_x,y,z) = A \sin\left(\frac{\ell\pi L_x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right) \sin\left(\frac{n\pi z}{L_z}\right)$$

The only nontrivial way this can equal zero is if $\sin(\ell\pi L_x/L_x) = 0$. Thus ℓ must be an integer. Similarly, the boundary conditions for y and z are satisfied provided m and n are integers.