speed. Thus we have defined three measures of average particle speed: the most probable, the average, and the root mean square or rms (Exer. 2, Exmp. 2, and Exer. 4, respectively).

$$V_{mp} = \sqrt{2RT} \qquad \hat{V} = \sqrt{\frac{8RT}{\pi}} \qquad V_{rms} \equiv \left(\widehat{V^2}\right)^{\frac{1}{2}} = \sqrt{3RT} \qquad \text{(IIIE-3,5a,b)}$$

These averages are not far apart numerically and are of identical form. Each is directly proportional to the square root of temperature and inversely proportional to the square root of particle molecular weight.

Exercise 5. For purposes of comparison determine V_{mp} and V_{rms} for the common, and not so common, constituents of the atmosphere at 300 K: nitrogen, N_2, water vapor, H_2O, helium, He, and gasoline (octane) fumes, C_8H_{18}.

The various average speeds specified by eqn. IIIE-3 provide one measure of the character of the molecular speed distribution. A second approach is the specification of the fraction of particles that have speeds within a certain range. Table IIIE-1 includes a tabulation of the probability integral up to specified dimensionless speeds, $x^* = V^*/\sqrt{2RT}$. The fraction of particles with speeds between zero and V^* is

$$P(0 \to V^*) = \int_0^{V^*} \rho(V)\,dV = \frac{4}{\sqrt{\pi}} \int_0^{V^*/\sqrt{2RT}} x^2\, e^{-x^2}\, dx \qquad \text{(IIIE-6)}$$

Use of this function is illustrated in the following exercises.

Exercise 6. Determine the percentage of oxygen molecules at 600 K whose speeds are above 1000 m/s.

Exercise 7. What fraction of gaseous molecules travel at speeds between V_{mp} and V_{rms}?

The procedure used to derive the speed distribution of molecular gas particles can also be applied to the free electrons of a metallic solid. At normal temperatures free electrons are distributed over their energy levels in a manner that is very similar to their distribution at absolute zero, $T \ll \theta_F$ (see Fig. IIID-4). Therefore, a good approximation to the free electron speed distribution at all practical temperatures is the distribution they exhibit at absolute zero.

Example 3. Exmp. 8 of Unit IIID used the distribution of free electrons at absolute zero to determine the value of the Fermi level at absolute zero, ε_F°. Transform this energy distribution to obtain a speed distribution for free electrons at zero temperature.

At absolute zero the number of free electrons with energy ε is

$$n_{el}(\varepsilon)\Big|_{\substack{T=0\\ \varepsilon<\varepsilon_F^\circ}} = g_{el}(\varepsilon) = 4\pi V\left(\frac{2m_e^*}{h^2}\right)^{3/2} \varepsilon^{\frac{1}{2}} \quad \text{and} \quad n_{el}(\varepsilon)\Big|_{\substack{T=0\\ \varepsilon>\varepsilon_F^\circ}} = 0$$

Also the total number of electrons was shown to be

$$N_{el} = \frac{8\pi V}{3}\left(\frac{2m_e^*}{h^2}\right)^{3/2} \varepsilon_F^{\circ\,3/2}$$