$$\rho(V) = \frac{4\pi V^2 \, e^{-V^2/2RT}}{(2\pi RT)^{3/2}} \qquad \text{(IIIE-2)}$$

Exercise 1. Express the velocity distribution in a spherical coordinate representation of velocity space ($0 < V < \infty$, $0 < \theta < \pi$, $0 < \phi < 2\pi$) and integrate over all directions to obtain the particle speed distribution.

The speed distribution is sketched in Fig. IIIE-2. It is not a symmetric curve and its maximum, *the most probable speed*, is not zero.

$$V_{mp} = \sqrt{2RT} \qquad \text{(IIIE-3)}$$

Exercise 2. Differentiate the speed distribution, eqn. IIIE-2, to derive eqn. IIIE-3 for the most probable speed.

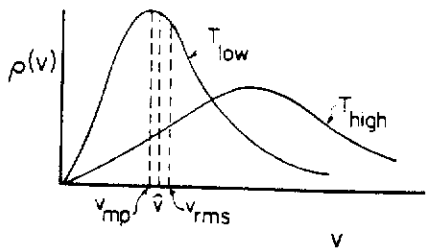

Fig. IIIE-2. Speed Distribution in a Molecular Gas

Exercise 3. We had earlier noted that the most probable velocity is zero, Fig. IIIE-1. We now find that the most probable speed is not zero, Fig. IIIE-2. How can the highest probability be simultaneously a stationary and a moving particle?

The mean of any function of speed, $\phi(V)$, is defined by

$$\widehat{\phi(V)} \equiv \int_0^\infty \phi(V)\rho(V)dV \qquad \text{(IIID-4)}$$

This definition is applied to determine the average speed as well as the average of the square, cube, and other multiples of the speed that are of physical interest.

Example 2. Use the definition of the mean and the integrals of Table IIIE-1 to determine the average speed of independent molecules of a molecular gas.

The average speed is defined by

$$\hat{V} \equiv \int_0^\infty V\rho(V)dV$$

To evaluate the required integral, we express it in terms of the dimensionless argument ($x^2 = V^2/2RT$, $dx = dV/\sqrt{2RT}$) and use Table IIIE-1[7]

$$\hat{V} \equiv \frac{4\pi}{(2\pi RT)^{3/2}} \int_0^\infty V\, V^2\, e^{-V^2/2RT} dV = \frac{4\pi}{(2\pi RT)^{3/2}} (2RT)^2 \int_0^\infty x^3\, e^{-x^2}\, dx = 4\left(\frac{2RT}{\pi}\right)^{\frac{1}{2}} \left(\frac{1}{2}\right) = \left(\frac{8RT}{\pi}\right)^{\frac{1}{2}}$$

Exercise 4. Determine the average kinetic energy of the particles in a molecular gas, $\hat{\epsilon}_{ke} = m\,\widehat{V^2}/2$. Verify that the result is the same as the translational mode's contribution to the internal energy of perfect gases.

In Exer. 4 the average of the square of the speed was determined. The square root of this parameter, the *root mean square speed* provides an energy-based measure of average particle

[7]The parameter $x(= V/\sqrt{2RT})$ is often called the molecular speed ratio.