$$\ln W_i^{BE} = \sum_j [(g_j+n_{j,i}) \ln (g_j+n_{j,i}) - (g_j+n_{j,i}) - g_j \ln g_j + g_j - n_{j,i} \ln (n_{j,i}) + n_{j,i}]$$

Collecting terms with common coefficients, we have

$$\ln W_i^{BE} = \sum_j g_j \ln \left(\frac{g_j+n_{j,i}}{g_j}\right) + n_{j,i} \ln \left(\frac{g_j+n_{j,i}}{n_{j,i}}\right)$$

This gives eqn. IIIB-B3 when multiplied by k, i.e., $S_i^{BE} = k \ln W_i^{BE}$.

Exercise B3. Derive S_{eq}^{BE}, eqn. IIIB-6, in terms of β by substituting eqn. IIIB-B4 into eqn. IIIB-B3 and simplifying.

Exercise B4. Partially differentiate eqn. III-6 to show that $(\partial S_{eq}^{BE}/\partial U)_{N,V} = \frac{1}{T} = k\beta$.

II. THE GIBBS EQUATION FOR FERMIONS AND BOSONS

The Gibbs equation was derived in Section III of the text by differentiating the entropy and recognizing the microscopic expression for reversible work, $đW_{rev}^i = \sum_j n_{j,i} d\varepsilon_j(V)$. This same procedure can be followed for both Fermions and Bosons. We demonstrate the procedure for Fermions in Exmp. B5 and ask that you complete the same derivation for Bosons, Exer. B5.

Example B5. Differentiate eqn. IIIB-B5 at fixed N and simplify to obtain the Gibbs equation.

The development is very similar to Exmp. B4. We differentiate term by term

$$dS^{FD}\Big|_N = \frac{dU}{T} - \frac{UdT}{T^2} + Nkd\alpha + k \sum_j \left(\frac{g_j}{e^{\alpha+\varepsilon_j(V)/kT}+1}\right) \left(-d\alpha \frac{-1}{kT} \frac{d\varepsilon_j(V)dV}{dV} + \frac{\varepsilon_j(V)dT}{kT^2}\right)$$

Then recognizing $n_{j,mp}^{FD}$, we have

$$dS^{FD}\Big|_N = \frac{dU}{T} - \left(\frac{U}{T^2} + \frac{k \sum n_{j,mp}^{FD} \varepsilon_j(V)}{kT^2}\right) dT + \left(Nk - k \sum n_{j,mp}^{FD}\right) d\alpha - \frac{1}{T} \sum_j n_{j,mp}^{FD} d\varepsilon_j(V)$$

or

Ans. B2. Differentiate the terms of eqn. IIIB-B3, to obtain

$$d\left(\frac{S_i^{BE}}{k}\right) = \sum_j \left[\frac{g_j}{g_j+n_{j,i}} + \ln\left(\frac{g_j+n_{j,i}}{n_{j,i}}\right) + \left(\frac{n_{j,i}}{g_j+n_{j,i}}\right) - \frac{n_{j,i}}{n_{j,i}}\right]\delta n_{j,i} = \sum_j \ln \left(\frac{g_j+n_{j,i}}{n_{j,i}}\right)\delta n_{j,i}$$

Substitute into eqn. IIIB-7 for

$$\sum_j \ln \left[\left(\frac{g_j+n_{j,i}}{n_{j,i}}\right) - \alpha - \beta\varepsilon_j(V)\right] \delta n_{j,i} = 0 \quad \text{or} \quad \ln \left(\frac{g_j+n_{j,i}}{n_{j,i}}\right) = \alpha+\beta\varepsilon_j(V)$$

whose antilog is eqn. IIIB-B4