The partition function is applicable to Boltzons and to Bosons and Fermions in the indistinguishable limit. It appears in each of the above expressions. The effort required to evaluate properties is greatly reduced if we formulate all properties in terms of the partition function and its derivatives.[2]

$$\left[\frac{\partial Z(T,\eta\bar{v})}{\partial T}\right]_{\bar{v}} = \sum_{j=1}^{J} g_j\left[\frac{\varepsilon_j(\eta\bar{v})}{kT^2}\right]e^{-\varepsilon_j(\eta\bar{v})/kT} = -\frac{1}{kT^2}\sum_{j=1}^{J} g_j\varepsilon_j(\eta\bar{v})e^{-\varepsilon_j(\eta\bar{v})/kT}$$

and

$$\left[\frac{\partial Z(T,\eta\bar{v})}{\partial \bar{v}}\right]_{T} = \sum_{j=1}^{J} g_j\left[-\frac{1}{kT}\frac{\partial\varepsilon_j(\eta\bar{v})}{\partial\bar{v}}\right]e^{-\varepsilon_j(\eta\bar{v})/kT} = -\frac{\eta}{kT}\sum_{j=1}^{J} g_j\left[\frac{d\varepsilon_j(\eta\bar{v})}{d(\eta\bar{v})}\right]e^{-\varepsilon_j(\eta\bar{v})/kT}$$

With these functions we can recognize the correspondence of properties to the partition function from their definitions.

1. Internal Energy is defined as

$$\bar{u} = \sum_{j=1}^{J} n_j\varepsilon_j(\eta\bar{v}) = \frac{N_A}{Z(T,\eta\bar{v})}\sum_{j=1}^{J} g_j\varepsilon_j(\eta\bar{v})\, e^{-\varepsilon_j(\eta\bar{v})/kT}$$

Examining the form of $(\partial Z/\partial T)_{\bar{v}}$, we see that $\bar{u} = (N_A kT^2/Z)(\partial Z/\partial T)_{\bar{v}}$ or

$$\bar{u} = \frac{\bar{R}T^2}{Z}\left[\frac{\partial Z(T,\eta\bar{v})}{\partial T}\right]_{\bar{v}} = \bar{R}T^2\left[\frac{\partial \ln Z(T,\eta\bar{v})}{\partial T}\right]_{\bar{v}} \qquad \text{(IIIC-3)}$$

2. Entropy. With the development of eqn. IIIC-3 the entropy can be written in terms of the partition function

$$\bar{s}^{MB} = \bar{R}T\left[\frac{\partial \ln Z(T,\eta\bar{v})}{\partial T}\right]_{\bar{v}} + \bar{R}\ln Z(T,\eta\bar{v}) \qquad \text{(IIIC-4a)}$$

$$\bar{s}^{il} = \bar{R}T\left[\frac{\partial \ln Z(T,\eta\bar{v})}{\partial T}\right]_{\bar{v}} + \bar{R}\ln\left[\frac{Z(T,\eta\bar{v})}{\eta N_A}\right] + \bar{R} \qquad \text{(IIIC-4b)}$$

3. Pressure was defined in terms of microscopic parameters in Unit IIIB (see footnote 15)

$$p = -\sum_{j=1}^{J} n_j\frac{d\varepsilon_j(V)}{dV} = \frac{\eta N_A}{Z}\sum_{j=1}^{J} g_j\left[\frac{d\varepsilon_j(\eta\bar{v})}{d(\eta\bar{v})}\right]e^{-\varepsilon_j(\eta\bar{v})/kT}$$

Examining the form of $(\partial Z/\partial\bar{v})_T$, we see that

$$p = \frac{\bar{R}T}{Z}\left.\frac{\partial Z}{\partial\bar{v}}\right|_T = \bar{R}T\left[\frac{\partial \ln Z(T,\eta\bar{v})}{\partial\bar{v}}\right]_T \qquad \text{(IIIC-5)}$$

4. Enthalpy. Substitution of eqns. IIIC-3 and IIIC-5 into the definition of enthalpy yields

$$\bar{h} = \bar{u} + p\bar{v} = \bar{R}T\left[T\left(\frac{\partial \ln Z}{\partial T}\right)_{\bar{v}} + \bar{v}\left(\frac{\partial \ln Z}{\partial\bar{v}}\right)_T\right] \qquad \text{(IIIC-6)}$$

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[2] All derivatives are performed at a fixed number of moles. The subscript η is implicit.