$$n_j^{MB} = \frac{Ng_j e^{-\varepsilon_j(V)/kT}}{Z(T,V)} \qquad \text{where} \qquad Z(T,V) = \sum_j g_j e^{-\varepsilon_j(V)/kT} \qquad \text{(IIIB-11,12)}$$

In eqn. IIIB-11, and in the rest of the unit, the subscript, mp, is omitted. It is implied in the expression for the distribution function. Similarly, we will omit the subscript, eq, in the expression for entropy.

$$S^{MB} = \frac{U}{T} + Nk \ln Z(T,V) \qquad \text{(IIIB-13)}$$

The use of the distribution function can be illustrated by example. Although most physical systems have an infinite number of states and levels (e.g., the particle in a box, Table IIIA-1), it is simpler to consider a system with only a few levels. A physical example that serves this purpose is a laser. The lasing states are a few levels in an infinite array of available excitation levels, one of which has a long time constant for decay to equilibrium relative to the others. For processes performed in times small in comparison to the decay time, this metastable state and the ground state can be considered isolated.

Example 3. The operation of a ruby laser can be analyzed as an isolated system of electrons existing in two levels— the ground level, $^4A_2$, and the metastable level, ^2E. Operating at liquid nitrogen temperatures, 77 K, a particular ruby crystal has the following characteristics:[13]

$$\varepsilon_1 = 0, \quad g_1 = 6.40 \cdot 10^{22}, \quad \varepsilon_2 = 2.88 \cdot 10^{-20} \text{ J/particle}, \quad g_2 = 3.20 \cdot 10^{21}$$

The total number of electrons trapped in this isolated subsystem is 10^{18}.

(a) Determine the number of electrons in each level of the subsystem when it is in equilibrium with the rest of the crystal (77 K). Determine the total energy of the subsystem under these conditions. (b) A power source is used to add energy to the lasing system (i.e., to pump it) until the number of electrons in the upper level is $n_2 = 5.00 \cdot 10^{16}$ Find the amount of energy added.

(a) We first compute the partition function.

$$Z = \sum_j g_j e^{-\varepsilon_j/kT} = g_1 e^{-\varepsilon_1/kT} + g_2 e^{-\varepsilon_2/kT}$$

$$= 6.40 \cdot 10^{22} + 3.20 \cdot 10^{21} \cdot \exp\left(\frac{-2.88 \cdot 10^{-20} \text{ J/part}}{1.38 \cdot 10^{-23} \text{ J/(part K)} \cdot 77 \text{ K}}\right)$$

$$= 6.40 \cdot 10^{22} + 3.20 \cdot 10^{21} \cdot \exp(-27.1) \approx 6.40 \cdot 10^{22}$$

[13]The energy change, $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$, is correct for the lasing states of a ruby crystal. But ε_1, g_1 and g_2 were selected arbitrarily for simplicity.