(Example 4, continued) adiabatic work is performed on the system causing the energy levels to change to ($\varepsilon_1 = 2.30 \cdot 10^{-26}$ J/part, $\varepsilon_2 = 11.5 \cdot 10^{-21}$ J/part; g_1 and g_2 are unchanged). Determine the work done on the system and the new temperature.

The energy of the system uniquely determines the distribution since there are only two states

$$U = n_1\varepsilon_1 + n_2\varepsilon_2 = 0 + n_2\varepsilon_2; \qquad n_2 = \frac{U}{\varepsilon_2} = \frac{2.07 \cdot 10^{-17}\ \text{J}}{6.90 \cdot 10^{-21}\ \text{J/part}} = 3 \cdot 10^3\ \text{part}$$

This gives $n_1 = N - n_2 = (10-3) \cdot 10^3 = 7 \cdot 10^3$ part. Since the particles are distinguishable they are Boltzons. In equilibrium

$$n_1 = \frac{Ng_1 \exp(-\varepsilon_1/kT)}{Z} \qquad \text{or} \qquad Z = \frac{10^4\ \text{part} \cdot 5 \cdot 10^2\ e^{-0}}{7 \cdot 10^3\ \text{part}} = 714$$

Then using

$$n_2 = \frac{Ng_2 \exp(-\varepsilon_2/kT)}{Z}; \qquad -\frac{\varepsilon_2}{kT} = \ln\left(\frac{n_2 Z}{Ng_2}\right)$$

or

$$T = \frac{-\varepsilon_2}{k \ln\left(\frac{n_2 Z}{Ng_2}\right)} = \frac{-6.90 \cdot 10^{-21}\ \text{J/part}}{1.38 \cdot 10^{-23}\ \text{J/(part K)} \cdot \ln\left(\frac{3 \cdot 10^3\ \text{part} \cdot 714}{10^4\ \text{part} \cdot 10^3}\right)} = 324\ \text{K}$$

Ans. 2. (a) $N = \sum_j n_{j,i} = 6430 + 2610 + 960 = 10^4$,

$U = \sum_j n_{j,i}\varepsilon_j = 0 + 2610\ \text{part} \cdot 2.76 \cdot 10^{-21}\ \text{J/part} + 960\ \text{part} \cdot 4.14 \cdot 10^{-21}\ \text{J/part} = 1.12 \cdot 10^{-17}\ \text{J}$

(b) If the system is in equilibrium, then

$$n_{j,i} = \frac{Ng_j \exp(-\varepsilon_j/kT)}{Z}; \qquad \text{or} \qquad 6430 = \frac{10^4 \cdot 10^4\ e^{-0}}{Z};$$

and

$$2610 = \frac{10^4 \cdot 3.00 \cdot 10^4}{Z} \exp\left[\frac{-2.76 \cdot 10^{-21}\ \text{J/part}}{1.38 \cdot 10^{-23}\ \text{J/(part K)}T}\right] = \frac{3 \cdot 10^8}{Z} \exp\left(\frac{-200\ \text{K}}{T}\right)$$

$$960 = \frac{10^4 \cdot 3.00 \cdot 10^4}{Z} \exp\left[\frac{-4.14 \cdot 10^{-21}\ \text{J/part}}{1.38 \cdot 10^{-23}\ \text{J/(part K)}T}\right] = \frac{3 \cdot 10^8}{Z} \exp\left(\frac{-300\ \text{K}}{T}\right)$$

These are 3 equations in two unknowns (Z and T). We can solve for Z with the n_1 condition; $Z = 1.56 \cdot 10^4$. Then solve for T with the n_2 condition; T = 100 K. Then test to see if these answers are consistent with the n_3 condition; they are. The macrostate is an equilibrium macrostate. If $n_2 = 3000$, then

$U = \sum n_{j,i}\varepsilon_j = 1.12 \cdot 10^{-17}\ \text{J} = 0 + 3000 \cdot 2.76 \cdot 10^{-21}\ \text{J} + n_3 \cdot 4.14 \cdot 10^{-21}$ J/part gives

$n_3 = 700$ and $n_1 = 10^4 - 3700 = 6300$. This is not an equilibrium state. It does not obey eqn. IIIB-11. Other equilibrium states for this system are

$$(n_{1,eq} = 6429,\ n_{2,eq} = 2613,\ n_{3,eq} = 958) \text{ and } (n_1 = 6431,\ n_2 = 2607,\ n_3 = 962).$$

These still satisfy eqn. IIIB-11 with $T = T_{eq} = 100$ K, to three-place accuracy.