substituted for the speed of light, c, and that the multiple two (introduced to account for the two directions of polarization of transverse light waves) becomes three (accounting for both longitudinal waves and the two directions of polarization of the transverse waves).[14] Thus, in terms of frequency the phonon degeneracy is $g_{phn}(\nu) = 12\pi V\nu^2/c_s^3$.

Exercise 10. Is the characteristic temperature that prescribes the frequency spacing of phonons, θ_{phn}, smaller or larger than that for photons, eqn. IIID-2? Draw a conclusion with respect to the significance of the phonon frequency spacing.

In our consideration of electromagnetic waves we focused on the frequency (or wavelength) distribution of photon energy (or emission). In the case of acoustical waves, we are relatively unconcerned about the energy distribution. Instead, we proceed directly to total energy by integrating the energy distribution over the *allowed phonon frequencies*. The fact that there is a limit on phonon, but not photon, frequencies is the principal difference between these otherwise very similar distributions. There is no upper limit on the energy levels accessible to photons. However, the total number of energy states accessible to the phonons is limited by the vibrations of their source. The N atomic lattice points can each vibrate in three independent directions. Thus, there can be only a total of 3N states accessible to the lattice vibration-generated phonon gas. This limit is imposed by using it to stipulate a *maximum Debye cutoff frequency*, ν_D

$$3N = \int_0^{\nu_D} g_{phn}(\nu)d\nu = \frac{12\pi V}{c_s^3}\int_0^{\nu_D} \nu^2 d\nu = \frac{4\pi V}{c_s^3}(\nu_D^3) \qquad \text{(IIID-11a)}$$

The cutoff frequency is sufficient to determine the contribution of lattice vibrations to crystalline properties. That being the case, eqn. IIID-11a is a particularly important relationship. It defines ν_D in terms of the sound speed and particle density of the substance.[14] (Particle density is determinable from the density and molecular weight of the crystal, $N/V = \rho N_A/M$). Following normal practice we express this microscopic energy limit in units of temperature

$$\theta_D \equiv \frac{h\nu_D}{k} = \frac{h\,c_s}{k}\left(\frac{3N}{4\pi V}\right)^{1/3} = \frac{h\,c_s}{k}\left(\frac{3\rho N_A}{4\pi M}\right)^{1/3} \qquad \text{(IIID-11b)}$$

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[14]Sound is transmitted through crystals by means of lattice vibrations. Thus sound speed is related to the elastic constants, see Ref. 5, pg. 287. The separate longitudinal and transverse sound speeds are

$$c_\ell = \left[\frac{3(1-\mu)}{\rho\kappa(1+\mu)}\right]^{1/2} \qquad c_t = \left[\frac{3(1-2\mu)}{2\rho\kappa(1+\mu)}\right]^{1/2}$$

where κ is the isothermal compressibility and μ is Poisson's ratio. Representative values for a few substances are listed in Table ID-2. The average sound speed is defined as

$$\frac{3}{c_s^3} = \left(\frac{1}{c_\ell^3} + \frac{2}{c_t^3}\right)$$