where $x_{max} = hc/\lambda_{max}kT$. Solving by trial and error we obtain, $x_{max} = 4.9651$. Thus the condition for the wavelength maximum is

$$\lambda_{max}T = \frac{hc}{x_{max}k} = \frac{6.625 \cdot 10^{-34}\text{J s/part} \cdot 2.998 \cdot 10^{8}\text{m/s}}{4.9651 \cdot 1.380 \cdot 10^{-23}\text{J/(part K)}} = 2.898 \cdot 10^{-3}\text{m K} = 2898\ \mu\text{m K}$$

The position of the maximum in Planck's Law is developed in Exmp. 1. The resulting expression is called *Wein's Law.*[9]

$$\lambda_{max}T = 2898\ \mu\text{m K} \qquad \text{(IIID-6)}$$

The maximum shifts to lower wavelength as the temperature level of the radiation is increased.

Exercise 4. Determine the wavelength of maximum emission for objects at 300, 3000, and 30,000 K.

Solar radiation is emitted from the sun's surface at about 5600 K. Thus, the maximum of available "natural" light on earth is at 0.52 μm. It is surely no coincidence that this is the middle of the visible spectrum. The eye is capable of detecting radiation in the wavelength range from about 0.4 to 0.7 μm.

Exercise 5. Alien visitors from a distant solar system have very limited sight on earth. Their eyes are sensitive in the wavelength range 0.2 to 0.3 μm. Estimate the emission temperature of their "sun."

Planck's Law gives the wavelength distribution of thermal radiation. The total energy density is obtained by integrating over all wavelengths.

$$u_v(T) = \int_0^\infty u_{v\lambda}(T)\ d\lambda \qquad \text{(IIID-7a)}$$

This process is represented graphically as the area under the distribution, see Fig. IIID-1.

Example 2. Substitute Planck's Law into eqn. IIID-7a and express the integral in terms of the dimensionless variable, $x \equiv (hc/\lambda kT)$.

Direct substitution gives

$$u_v(T) = \int_0^\infty u_{v\lambda}(T)d\lambda = 8\pi hc \int_0^\infty \frac{d\lambda}{\lambda^5[e^{(hc/\lambda kT)} - 1]}$$

Then using

$$x = \frac{hc}{\lambda kT} \qquad dx = \frac{-hc}{\lambda^2 kT}\,d\lambda \qquad \lambda = 0,\ x \Rightarrow \infty;\quad \lambda \Rightarrow \infty,\ x \Rightarrow 0$$

[9]The normal unit of wavelength is the micrometer, μm. In this context it is often abbreviated to micron, μ.

Ans. 3.

$$\theta_{pht|V=1mm^3} = \frac{6.625 \cdot 10^{-34}\text{J s/wave} \cdot 2.998 \cdot 10^{8}\text{m/s}}{2 \cdot 1.380 \cdot 10^{-23}\text{J/wave K} \cdot [10^{-9}\text{m}^3]^{1/3}} = 7.196\ \text{K}, \qquad \Theta_{pht|1m^3} = 7.196 \cdot 10^{-3}\text{K}$$