$$F_r = F_{thr,r} - (p_e - p_o)A_e = \dot{m}\hat{V}_e \qquad \text{(VD-2b)}$$

The forces on the engine, F_j or F_r, have been separated into two components—the exterior pressure force and the thrust exerted on the airframe.[4] The rate at which the engine does work in steady, level (or gravity free) flight is the product of its propulsive force and the forward velocity, $\dot{W}_j = F_j\hat{V}_o$ or $\dot{W}_r = F_r\hat{V}_o$. The two force components also separate the engine's power into two components. An underexpanded nozzle $p_e > p_o$, adds to the thrust developed by the engine to move the airframe forward. This thrust component is part of the engine's output. (A greater net output could be obtained by using a fully expanded nozzle that would achieve an optimal exit velocity corresponding to the exit pressure.) Independent of the nozzle's degree of expansion, in steady level flight, the thrust required to drive the airframe forward is equal to the drag force, $F_{thr} = F_D$. Under these conditions the power requirement to drive the airframe at speed $\hat{V}_o$ is $\dot{W}_{thr} = F_{thr}\hat{V}_o$.[5] Thus the work *output* per mass of flow for a jet or rocket engine in steady level flight is[6]

$$|w_{s,j}| = \hat{V}_o(\hat{V}_e - \hat{V}_o) \qquad |w_{s,r}| = \hat{V}_o\hat{V}_e \qquad \text{(VD-3a,b)}$$

We noted previously that the output of a propulsion system, relevant to its thermal efficiency, is the increase in kinetic energy that it produces in its working media relative to the engine.[7]

$$\Delta ke_j = \frac{(\hat{V}_e^2 - \hat{V}_o^2)}{2} \quad \text{or} \quad \Delta ke_r = \frac{\hat{V}_e^2}{2}$$

A comparison of the expressions for engine output shows that they are not equal. A portion of the change in relative kinetic energy developed by the engine is not converted to propulsion power. The loss equals the change in kinetic energy that the gas experiences relative to the ground; that is, $\Delta ke_{loss} = \Delta ke_{abs,j} = (\hat{V}_e - \hat{V}_o)^2/2$ or $\Delta ke_{abs,r} = [(\hat{V}_e - \hat{V}_o)^2 - \hat{V}_o^2]/2$. This loss is the basis for the definition of propulsive efficiency. It rates the proportion of propulsive power that is converted to thrust work.

$$\eta_p \equiv \frac{|w_s|}{|\Delta ke|} = \frac{|w_s|}{|w_s| + |\Delta ke_{abs}|} \qquad \text{(VD-4)}$$

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[4]The control surface for the analysis of the engine is taken sufficiently upstream that the pressure force on the entrance is p_o. Thus only the exit pressure differential appears.

[5]The energy analysis approach of Exmp. 1 and Exer. 1 did not separate the two power components. It listed only the power requirement to drive the aircraft forward.

[6]We continue to ignore the contribution of the fuel to the exit mass flow. A complete form of eqn. VD-3a is

$$|w_{s,j}| = \hat{V}_o[(1 + r_{f/a})\hat{V}_e - \hat{V}_o]$$

[7]$\Delta ke_r = \hat{V}_e^2/2$ incorporates the assumption that the propellant enters the rocket engine with a negligible relative velocity.