Values of the *Debye Temperature*, θ_D, of several solids are listed in Table IIID-1 at the conclusion of this unit.

Exercise 11. Use the listed values of θ_D to infer the sound speed in iron and lead.

The implication of the cutoff frequency is that no phonons exist at frequencies above ν_D. The limit is imposed on the distribution by specifying the degeneracy as

$$g_{phn}(\nu) = \frac{12\pi V}{c_s^3}\nu^2 \qquad 0 \le \nu \le \nu_D \qquad g_{phn}(\nu) = 0 \qquad \nu > \nu_D \tag{IIID-12a}$$

This gives the phonon distribution as

$$n_{phn}(\nu) = \frac{g_{phn}(\nu)}{(e^{h\nu/kT}-1)} \qquad n_{phn}(\nu) = \frac{12\pi V}{c_s^3}\frac{\nu^2}{(e^{h\nu/kT}-1)} \qquad 0 \le \nu \le \nu_D$$

$$n_{phn}(\nu) = 0 \qquad \nu > \nu_D \tag{IIID-12b}$$

With these expressions the energy of a phonon gas is readily formulated.

Example 5. Express the molar internal energy of a phonon gas, $\bar{u}_{phn}$, in terms of an integral over the dimensionless variable $x \equiv h\nu/kT$ and the dimensionless temperature ratio, $x_D = \theta_D/T$.

Phonon internal energy is defined by $U = \int_0^\infty (h\nu)n_{phn}(\nu)d\nu$. Substituting from eqn. IIID-12 and introducing the dimensionless variable

$$x = \frac{h\nu}{kT} \qquad dx = \frac{hd\nu}{kT} \qquad \text{when } \nu = 0,\ x = 0 \text{ and when } \nu = \nu_D,\ x = \frac{\theta_D}{T} = x_D$$

We have

$$U = \frac{12\pi V}{c_s^3}\int_0^{\nu_D} h\nu\left[\frac{\nu^2}{(e^{h\nu/kT}-1)}\right]d\nu + \int_{\nu_D}^{\infty} h\nu\cdot(0)\cdot d\nu = \frac{12\pi V}{c_s^3}\int_0^{\nu_D}\left(\frac{h\nu}{kT}\right)^3\frac{1}{(e^{h\nu/kT}-1)}\left(\frac{hd\nu}{kT}\right)$$

$$= 12\pi V\frac{(kT)^4}{(hc_s)^3}\int_0^{x_D}\frac{x^3dx}{(e^x-1)}$$

Then introducing the relations $\eta N_A = N$ and $\bar{R} = N_A k$, we obtain the result shown below as eqn. IIID-13a.

Exmp. 5 derived an expression for the internal energy of a phonon gas[15]

$$\bar{u}_{phn} = 9\,\bar{R}T\left(\frac{T}{\theta_D}\right)^3\int_0^{\theta_D/T}\frac{x^3dx}{e^x-1} \tag{IIID-13a}$$

[15]Eqns. IIID-13, 15, and 16 refer to a mole of lattice points in the crystal rather than a mole of phonons.

Ans. 10. $\theta_{phn} = h\,c_s/2kV^{1/3}$. The only change is that from the speed of light to the speed of sound. Thus, $\theta_{phn} \ll \theta_{pht}$. It took a combination of a very small enclosure at very low temperatures to achieve conditions under which $\theta_{pht} \sim T$. Thus, we always expect $\theta_{phn} \ll T$.