where ℓ, m = 1,2,3,..., and all possible combinations. These functions can be sketched in three dimensions.[37] The energy levels for a free particle within a rectangular plane are

$$\varepsilon_{\ell,m} = \frac{h^2}{8m^*}\left[\left(\frac{\ell}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2\right]$$

Let us consider the first few energy levels, wave functions, and probability distributions for a free particle confined within a plane.

The lowest energy level of this particle corresponds to the set of quantum numbers (1, 1) or ℓ = 1 and m = 1. This particle's wave function is

$$\psi_{1,1}(x,y) = A \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right)$$

and its probability distribution is

$$P_{1,1}(x,y) = A^2 \sin^2\left(\frac{\pi x}{L_x}\right) \sin^2\left(\frac{\pi y}{L_y}\right)$$

Fig. IIIA-A1 shows a sketch of these functions versus x and y. It is seen in the sketch of $P_{1,1}(x,y)/A^2$ that with this lowest energy it is probable that the particle will be at or near the center of the plane.

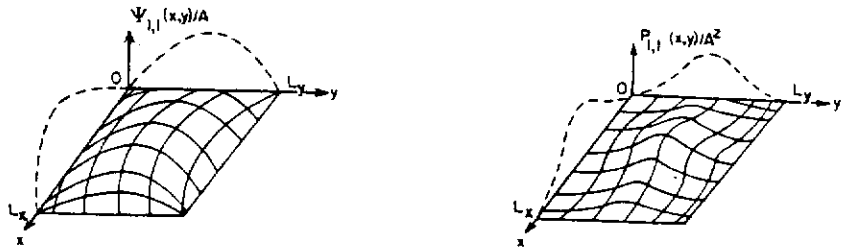

Fig. IIIA-A1. The Wave Function and Probability Distribution for a Particle of Minimum Energy in a Plane

Two higher energy levels for the particle correspond to the quantum number sets, ℓ = 1, m = 2, ℓ = 2, m = 1. For the quantum set (1, 2) the energy is

$$\varepsilon_{1,2} = \frac{h^2}{8m^*}\left[\left(\frac{1}{L_x}\right)^2 + \left(\frac{2}{L_y}\right)^2\right]$$

The wave function and probability distribution for the quantum set (1, 2) are

$$\psi_{1,2}(x,y) = A \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{2\pi y}{L_y}\right)$$

and

$$P_{1,2}(x,y) = A^2 \sin^2\left(\frac{\pi x}{L_x}\right) \sin^2\left(\frac{2\pi y}{L_y}\right)$$

[37]The three-dimensional box would require four-dimensional sketches.