The interested student can repeat the same steps for Bosons, Exers. IIIB-B1 and B2, below. In this case the appropriate expression for W_i is eqn. IIIB-7b. When this expression is substituted into eqn. IIIB-1 and simplified we obtain

$$S_i^{BE} = k\ \ell n\ W_i^{BE} = k \sum_j \left[g_j\ \ell n \left(\frac{g_j+n_{j,i}}{g_j}\right) + n_{j,i}\ \ell n \left(\frac{g_j+n_{j,i}}{n_{j,i}}\right)\right] \qquad \text{(IIIB-B3)}$$

Then one differentiates this result and solves eqn. IIIB-7 to obtain

$$n_{j,mp}^{BE} = \frac{g_j}{e^{\alpha+\beta\varepsilon_j(V)}-1} \qquad \text{(IIIB-B4)}$$

Exercise B1. Take the logarithm of eqn. IIIA-7b and use Stirling's approximation for the logarithm of a factorial, eqn. IIIB-A2 as well as the assumption that $g_j >> 1$ to derive eqn. IIIB-B3.

Exercise B2. Differentiate eqn. IIIB-B3 at fixed N and V. Then use Lagrange multipliers in the form of eqn. IIIB-7 to derive eqn. IIIB-B4.

The parameter β can be identified for both Fermions and Bosons using the same procedure as employed in the text for Boltzons. It requires the differentiation of the equilibrium entropy with respect to the energy holding the volume and number of particles constant, i.e., determine $(\partial S_{eq}/\partial U)_{N,V}$. We illustrate below for Fermions.

Example B3. Obtain a simplified expression for S_{eq}^{FD} by substitution of eqn. IIIB-B2 and elimination of $n_{j,mp}^{FD}$ from eqn. IIIB-B1.

We begin by writing eqn. IIIB-B1 in a more compact form.

$$S_{eq}^{FD} = k\left(\ell n\ W_{mp}^{FD}\right) = k \sum_j g_j\ \ell n \left(\frac{g_j}{g_j-n_{j,mp}^{FD}}\right) + k \sum_j n_j^{FD}\ \ell n \left(\frac{g_j}{n_{j,mp}} - 1\right)$$

Then arranging eqn. IIIB-B2 in forms suitable for substitution

$$\ell n \left(\frac{g_j}{n_{j,mp}^{FD}} - 1\right) = \alpha + \beta\varepsilon_j(V) \quad \text{and} \quad \frac{g_j}{g_j-n_{j,mp}^{FD}} = \frac{1}{1-\dfrac{1}{\exp[\alpha+\beta\varepsilon_j(V)]+1}} = 1+\exp[-\alpha-\beta\varepsilon_j(V)]$$

This gives

$$S_{eq}^{FD} = k \sum_j n_{j,mp}^{FD}[\alpha+\beta\varepsilon_j(V)] + k \sum_j g_j\ \ell n\ \{1+\exp[-\alpha-\beta\varepsilon_j(V)]\}$$

Since $N = \sum_j n_{j,mp}^{FD}$ and $U = \sum_j n_{j,mp}^{FD}\ \varepsilon_j(V)$, we obtain the relationship

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[21] Recall that g_j is a constant.