The preceding discussion is summarized in Fig. IIIE-4; it illustrates the form of the thermal conductivity of insulating solids. It also represents the phonon component of the total conductivity of an electrical conductor. The free electron contribution to the thermal conductivity of a *metallic conductor* can be discussed in a similar manner. For free electrons, eqn. IIIE-20 takes the form

$$K_{el} = \frac{\rho V_F \, c_{v,el} \, \lambda_{c,F}}{3}$$

where ρ is the crystalline density and $c_{v,el}$ is the free electron specific heat. In Unit IIID we found that $c_{v,el}[= (\pi^2/2) \cdot R \cdot (T/\theta_F)]$ was quite low, since only the electrons in the vicinity of the Fermi level are capable of being excited. Therefore, the velocity and mean free path of interest are those of the free electrons *at the Fermi level,* V_F and $\lambda_{c,F}$. Now consider the temperature dependence of $K_{el}(T)$. ρ and V_F are constant and the temperature dependence of $c_{v,el}(\propto T)$ is known. Therefore, the variations in K_{el} to be determined are governed by those in $\lambda_{c,F}$.

Free Electron Collision Processes. The important collision processes for free electrons are the same as those for phonons. We can, therefore, separate the form of K_{el} into low and high temperature regions.

In the *high temperature region,* $T > \theta_D$, the phonon density is sufficient that electron-phonon collisions predominate, $\lambda_{c,F} \simeq \lambda_{c,el-phn} \propto 1/n_{v,ph}$. The temperature dependence is determined by $n_{v,phn} \propto T$. Thus

$$K_{el}(T > \theta_D) = \frac{\rho V_F \, c_{v,el} \, \lambda_{c,el-p}}{3} \propto \frac{T}{T} \propto T^0$$

At high temperatures K_{el} is independent of temperature.

In the *low temperature region,* $T < \theta_D$, the phonon density is sufficiently reduced that $\lambda_{c,el-phn} > \lambda_{c,el-def} = \delta L$. Thus

$$K_{el}(T < \theta_D) = \frac{\rho V_F \, c_{v,el} \, \delta L}{3} \propto T$$

when the temperature is reduced to the point that defect collisions dominate, then K_{el} is linearly proportional to temperature. In very pure, carefully annealed conductors this transition is delayed to much lower temperatures. At low temperature levels the phonon density falls exponentially, $n_{v,phn} \propto \exp(-\theta_D/T)$. This casues K_{el} to increase until the defect scattering limit is reached, see Fig. IIIE-4.

We have noted that heat is carried simultaneously by phonons and electrons, Figs. IIIC-4 and 5. Their effects can be combined on the basis of

$$\frac{1}{K} = \frac{1}{K_{el}} + \frac{1}{K_{phn}} \qquad \text{(IIIE-23)}$$

Even though only a small percentage of the free electrons near the Fermi level participate in heat transport, so that $c_{v,el} > c_{v,phn}$, the speed of electron motion is so much faster