The minimum energy is determined by $\ell = 1$, $m = 1$. This energy is kinetic, $(1/2)m^*\hat{V}^2$. Intuitively we expect the minimum speed of the ball to equal zero. Treat the ball as a point particle of mass 250 g on a table of dimensions 1.2 m by 2.7 m. Calculate the minimum speed of the ball. Calculate the time required for the ball to move a minimum detectable macroscopic increment, 10^{-7} cm, at this speed.

Note to the Student. Unlike the previous section, thorough familiarity with the meaning of the terms defined and discussed in this section is essential to accomplish the objectives of this unit.

C. Energy States, Energy Levels, and Degeneracy

The solution of the Schroedinger equation for a particle in a field-free box resulted in an expression for the discrete energies of the particle, eqn. IIIA-4. The energy states of the particle are dependent on the dimensions of the box (L_x, L_y, L_z). The general concepts are more easily assimilated in terms of the simpler geometry of a cubic box. Within a cube, the discrete energy states of the particle can be written as a function of the mass of the particle and the volume of the cube.[8]

$$\varepsilon_{\ell,m,n} = \frac{h^2}{8m^*V^{2/3}}(\ell^2+m^2+n^2) = \varepsilon_{\ell,m,n}(m^*,V) = B_o(\ell^2+m^2+n^2) \qquad \text{(IIIA-5)}$$

To save writing the physical constant $(h^2/8m^*V^{2/3})$ is named B_o.

Every set of quantum numbers (ℓ,m,n) identifies a distinct energy state. Since the minimum value of each quantum number is one, the minimum energy magnitude (the first *level*) that a particle can possess is $3\,B_o$. The next higher energy level is $\varepsilon_2 = 6\,B_o$. There are three states in this level: (1,1,2), (1,2,1) and (2,1,1). Though the energy of the states is the same, they remain distinct. The behavior of particles residing in different states is the same, they remain distinct. The behavior of particles residing in different states of the same level is different.[9] The number of states in a level is called its *degeneracy*,

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[8]for a cubic box $L_x = L_y = L_z$, $L^3 = V$, and $L^2 = V^{2/3}$. Thus eqn IIIA-4 can be simplified as

$$\left(\frac{\ell^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{\ell^2}{L_z^2}\right) = \frac{1}{L^2}(\ell^2 + m^2 + n^2) = \frac{1}{V^{2/3}}(\ell^2 + m^2 + n^2)$$

[9]The only energy form available to a particle within a field-free box is kinetic energy. Kinetic energy (a scalar quantity) is proportional to particle speed, i.e., the magnitude of the particle velocity (a vector quantity). Total kinetic energy can be written as a sum of its x,y, and z components. For each energy magnitude there may exist more than one quantum set, ℓ,m, and n. These integers prescribe how the total kinetic energy of the state is distributed among the three coordinate directions. Since the energy of the particle is proportional to velocity, one might ask of our solution, "What is the velocity vector for a particle in quantum state (1,1,2)?" The stationary state Schroedinger equation cannot be used to specify this vector exactly. The vector can lie in any quadrant. Its z component energy will be twice its x or y component. To specify the direction of motion, we would require a solution of the complete, time-dependent equation. However, since the particle's energy is independent of its direction of motion, we can disregard considerations of direction in our studies.