Substituting eqn. IIIB-A2 into $\ln\left(W_i^{MB}\right)$, we obtain[19]

$\ln W_i^{MB} = N \ln N - N + \sum_j \left(n_{j,i} \ln g_j - n_{j,i} \ln n_{j,i} + n_{j,i}\right)$; then since $N = \sum_j n_{j,i}$

$$S_i^{MB} = kN \ln N + k \sum_j \left[n_{j,i} \ln g_j - n_{j,i} \ln n_{j,i}\right] \qquad \text{(IIIB-A3)}$$

The differential we require is obtained (N and g_j are constant) so,

$$d\left(S_i^{MB}\right) = kd\left(\ln W_i^{MB}\right) = \sum_j \left(\ln g_j - \frac{n_{j,i}}{n_{j,i}} - \ln n_{j,i}\right)\delta n_{j,i}$$

and using the fact that $\sum_j \delta n_{j,i} = 0$, we have

$$d\left(S_i^{MB}\right) = k \sum_j n \frac{g_j}{n_{j,i}} \delta n_{j,i} \qquad \text{(IIIB-A4)}$$

APPENDIX IIIB-B

EQUILIBRIUM CHARACTERISTICS OF FERMIONS AND BOSONS

The text of the Unit uses systems of Boltzons as its illustration in the development of equilibrium characteristics in microscopic terms. This appendix presents similar developments for Fermions and Bosons.

I. THE DISTRIBUTION FUNCTION FOR FERMIONS AND BOSONS

In this section we develop the distribution function for a System of Fermions and set up an exercise to determine the distribution function for Bosons. The distribution function for any system of independent particles is determined by solving eqn. IIIB-7.

$$d\left(\frac{S}{K} - \beta U - \alpha N\right) = d\left(\ln W_i\right) - \beta \sum_j \varepsilon_j(V)\delta n_{j,i} - \alpha \sum_j \delta n_{j,i} = 0 \qquad \text{(IIIB-7)}$$

The variation in the solution stems from the substitution of W_i for the different statistics described by eqns. IIIA-7. For Bosons and Fermions these are

$$W_i^{BE} = \prod_j \frac{(g_j+n_{j,i}-1)!}{(g_j-1)!\, n_{j,i}!}\ ; \qquad W_i^{FD} = \prod_j \frac{g_j!}{(g_j-n_{j,i})!\, n_{j,i}!} \qquad \text{(IIIA-7b,c)}$$

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[19]The use of eqn. IIIB-A2 for $\ln(n_{j,i}!)$ does not imply that no single $n_{j,i}$ is less than 10^2; but rather that the sum over j remains accurate since many $n_{j,i}$ are much greater than 10^2.