Note that if one is to use the first integral representation both, the degeneracy and the energy must be expressed in closed form as a function of the level index, $g_{p,\ell} \to g_p(\ell)$ and $\varepsilon_{p,\ell} \to \varepsilon_p(\ell)$. And if the second integral representation is to be used the degeneracy must be expressed as a function of the energy, $g_{p,\ell} \to g_p(\varepsilon_\ell)$.

Referring to the first of the preceding three possibilities, when the series is exact, e.g., the vibrational model, representation as a transcendental function is the preferred evaluation method. The last two methods are asymptotic temperature limits that exist for all modes. They are important not only for their analytic simplification, but also for their physical significance. *In its low-temperature limit a mode is unexcited.* It then contributes no more than a constant to properties. (The constant is frequently zero.) *In the high-temperature limit a mode is fully excited.* It is interesting to note that discreteness of the energy spectrum is central to the low-temperature asymptote (this is the quantum limit). But in the high-temperature limit energy can be viewed as continuous; this is the classical, or Newtonian limit.

Each mode has its own limits. (A high temperature to one mode may be low to another.) Therefore, the most significant characteristic for each mode is the parameter that discriminates between its high and low temperature limits. This parameter is identified by noting that the quantized energy levels can always be expressed in the form

$$\varepsilon_{p,\ell} = B_p f_p(\ell) \qquad \text{(IIIC-12)}$$

in which $f_p(\ell)$ is a dimensionless function of the level index, ℓ (or equivalently of the quantum numbers). Since the index is the positive integer series, $f_p(\ell)$ is a discontinuous function; it expresses the quantization of energy. The coefficient B_p has dimensions of energy. It contains the physical characteristics of the mode. We will find that the incremental changes in the quantum index function $f_p(\ell)$ are of order one, $f_p(\ell+1) - f_p(\ell) \sim O(1)$. Therefore, it is B_p that discriminates between the mode's high and low temperatures. The physical significance of B_p is more recognizable if its values are reported with dimensions of temperature. The *characteristic temperature* of a mode is defined as

$$\theta_p \equiv \frac{B_p}{k} \qquad \text{(IIIC-13)}$$

Exercise 1. The energy states for the particle in a box, translation of an independent particle, were found to be

$$\varepsilon_{tr}(\ell,m,n) = \frac{h^2}{8m^* V^{2/3}} (\ell^2 + m^2 + n^2)$$

where $\ell, m, n = 1,2,3, \ldots$. Identify the discrete dimensionless function of the quantum numbers $f_{tr}(\ell,m,n)$, the physical coefficient B_{tr} and the characteristic temperature θ_{tr} for this mode.

Since we will use values of the characteristic temperature to describe the properties of each substance in each mode, it is convenient to formulate the partition function and